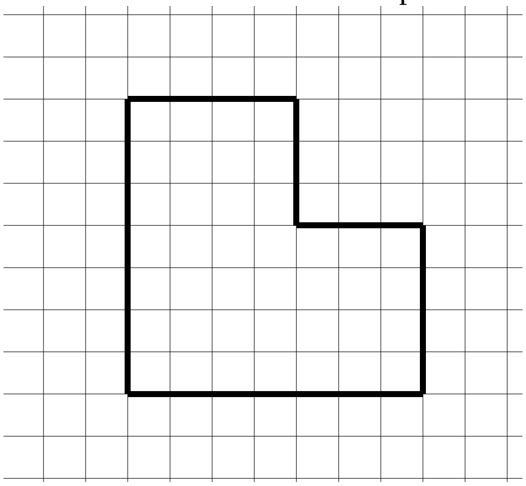
10.2a Class Activity: A Proof of the Pythagorean Theorem

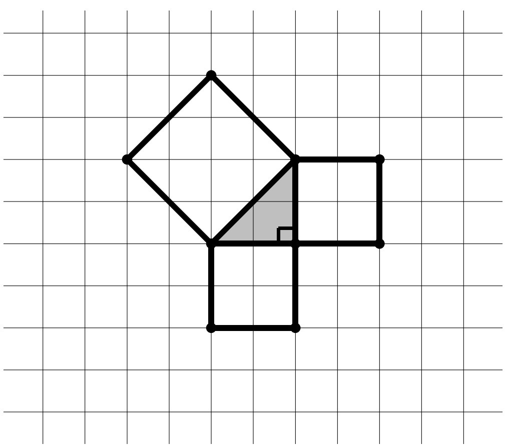
1. Find the area of the shape below. Each square on the grid has a side length of 1 unit.



In numbers 2 and 3, a right triangle is shown in gray. The shorter sides of a right triangle are referred to as **legs**. The longer side of the right triangle (the side opposite of the right angle) is called the **hypotenuse**.

Directions: Squares have been drawn adjacent to the sides of the right triangle. Find the area of each of the squares. Assuming each square on the grid has a side length of 1 unit. Write the areas inside each of the squares.

2.



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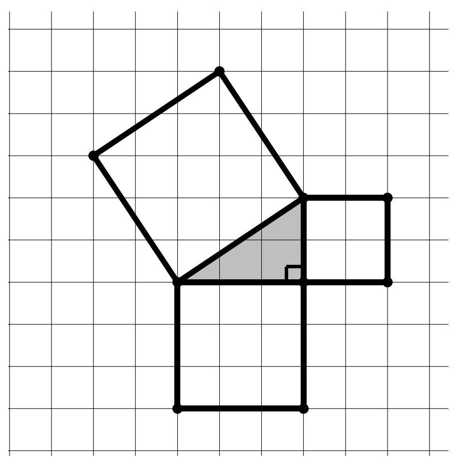
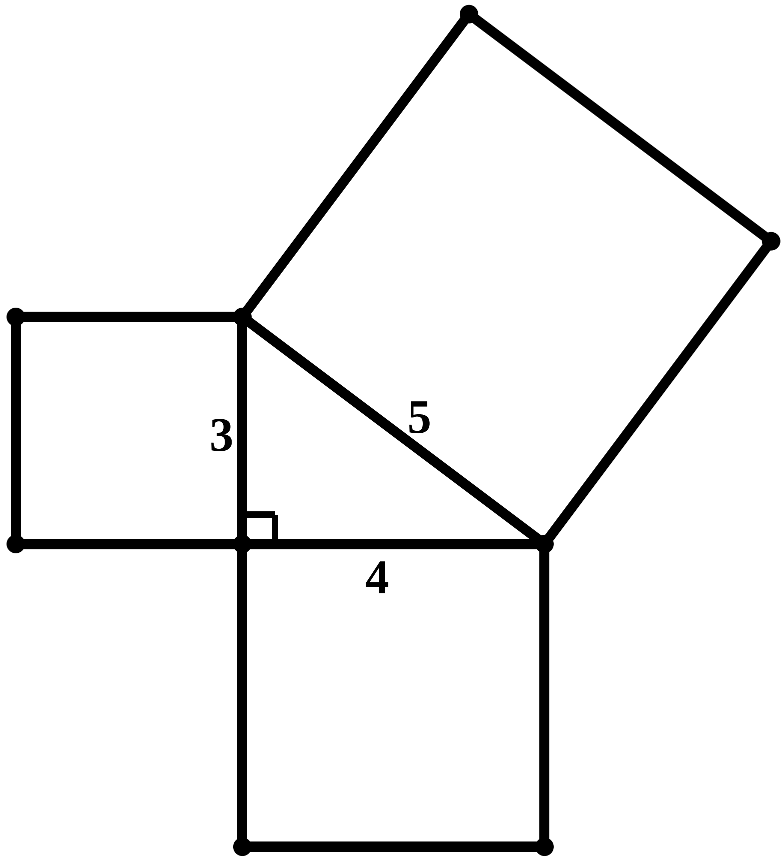


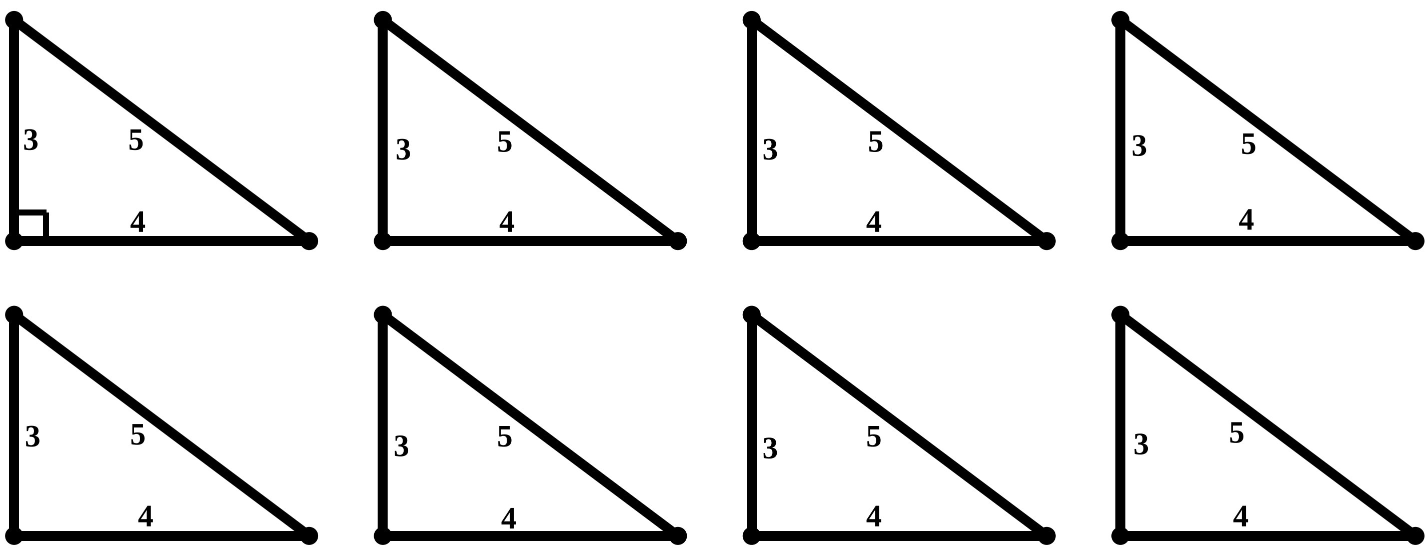
4. What do you notice about the relationship between the areas of the squares formed adjacent to the legs of a right triangle?

5. Below is a right triangle with side lengths 3, 4, and 5. Squares have been drawn adjacent to the sides of the right triangle.

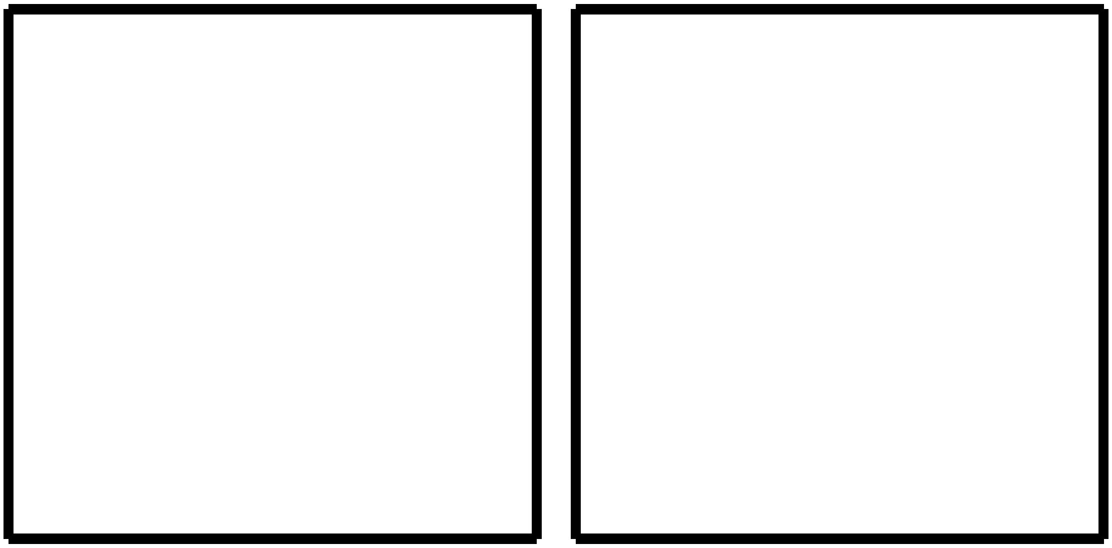


a. Find the area of each of the squares. Write the area inside each of the squares. Then, cut out the three squares very carefully.

b. Below are 8 copies of the original right triangle. Cut out the 8 triangles very carefully.



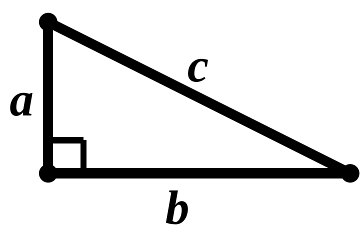
c. Below are two **congruent squares**. Since the squares are congruent, we know that their sides have the same length and subsequently they have the same area. Use your square with an area of 25 and four of the triangles from the previous page to cover one of the squares. Use your squares with areas 9 and 16 and four of the triangles from the previous page to cover the other square. Tape the pieces into place.

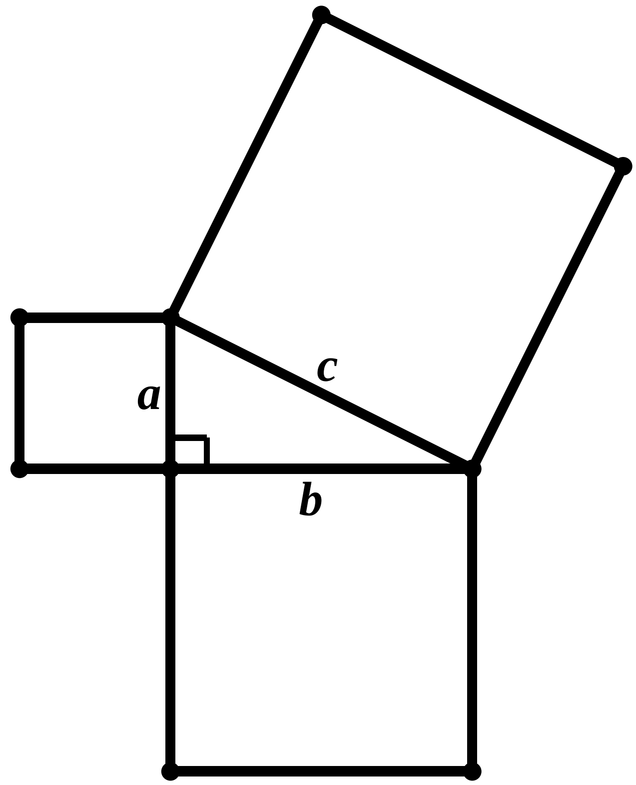


d. Use the large squares in part c) to explain the relationship you discovered in #2 – 4 between the squares formed adjacent to the sides of a right triangle.

6. In the previous problems, we saw that for specific triangles **the sum of the areas of the squares along the legs of the right triangle equals the area of the square along the hypotenuse of the triangle** by looking at several examples. Now, we want to show that this relationship holds true for ***any*** right triangle.

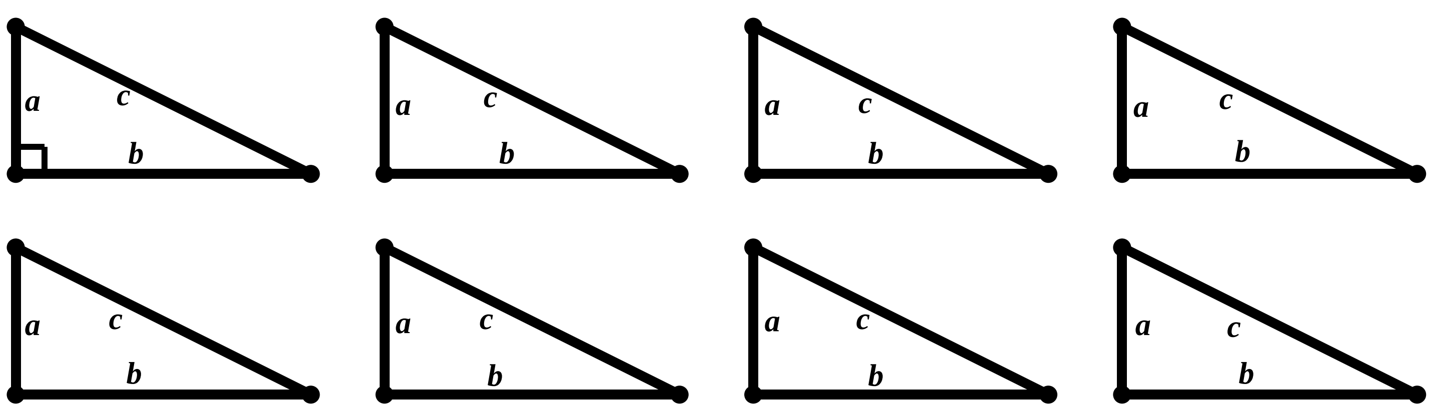
   Suppose you have a right triangle with any side lengths a, b, and c where a and b are the legs of the triangle and c is the hypotenuse of the right triangle as shown below. The squares have been drawn along the sides of the right triangle. Our goal is to show that $a^2 + b^2 = c^2$ is always true.



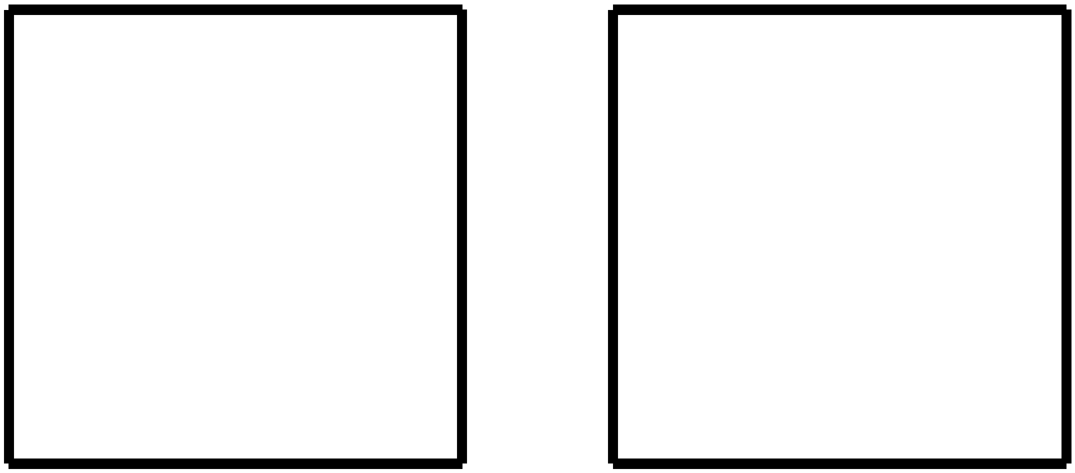


   a. Find the area of each of the squares adjacent to the sides of the right triangle. Write the areas inside each square.

   b. Cut out the squares formed on the sides of the triangle above as well as the 8 copies of the triangle with side lengths a, b, and c below.



c. Arrange the 3 squares and 8 triangles to cover the 2 squares shown below.



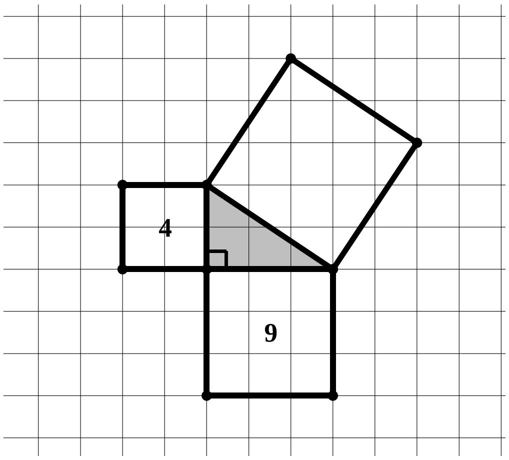
d. Using the picture above, show that **the sum of the areas of the squares adjacent to the legs of the right triangle equals the area of the square adjacent to the hypotenuse of the triangle for *<u>any</u>*** right triangle.

e. Conventionally, the leg lengths of a right triangle are denoted using the variables a and b and the hypotenuse of a right triangle is denoted using the variable c. State the relationship between the side lengths of a right triangle using the words **legs** and **hypotenuse**.

f. Write an **<u>equation</u>** that shows the relationship between the side lengths of a right triangle using a and b for the lengths of the legs and c for the length of the hypotenuse.

Directions: In each of the problems below, a right triangle is shown in gray. The squares along each of the three sides of the triangles have been drawn. The area of two of the squares is given. Determine the area of the third square. Write your answer in the square. Also find the side length of each square, write the sides lengths below each picture.

1.



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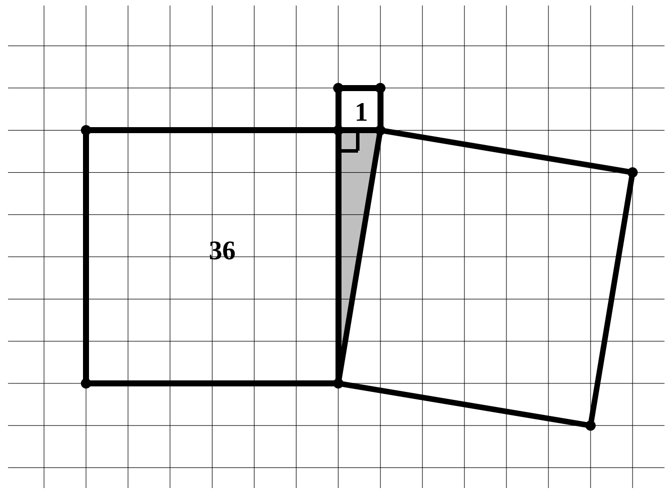

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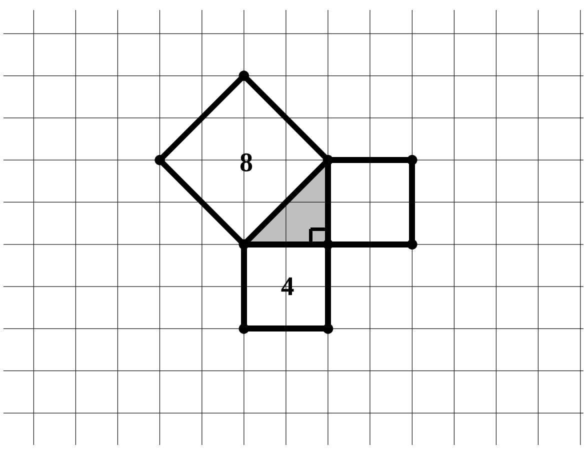

4.