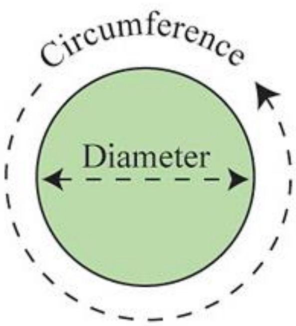
The Common Core State Standards for Eighth Grade Mathematics require students to learn the volume of three basic three-dimensional figures: **cylinders**, **cones** and **spheres**. Each of these relies on your previous knowledge of circles -- circumference, diameter, radius, pi, and area -- because each is circular.



The distance around a circle is the circumference, while the distance across a circle, through the center, is the diameter. The ratio of the circumference to the diameter is always the same: 3.14159265358979323… This is an irrational number because it never terminates and never repeats. It is called pi, a Greek letter whose symbol is π.

$$\pi = C/d$$

If we know the diameter of a circle, we can multiply it by π to find the circumference:

$$C = \pi d$$

Given the radius of a circle, we can double it (since a diameter is twice a radius) then multiply it by π to find the circumference:

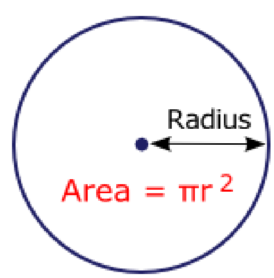
$$C = 2\pi r$$

If we are given the circumference, we can find the diameter by dividing it by π, with the radius being half that number:

$$\frac{C}{\pi} = d \quad or \quad \frac{C}{2\pi} = r$$

To find the area of a circle, use this formula:

$$A = \pi r^2$$



If you are given the diameter of a circle, you must first divide it by two to find the radius.

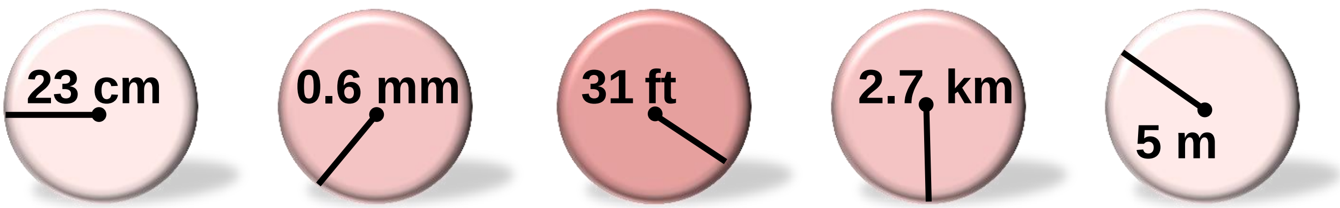
Note: circumference, radius and diameter would be expressed as cm, in, ft, m, yd, mi, km, etc., while area is expressed in square units such as cm^2, in^2, ft^2, yd^2 and mm^2.

CIRCLE PRACTICE PROBLEMS

Find the radius:

1. ________ 2. ________ 3. ________ 4. ________ 5. ________

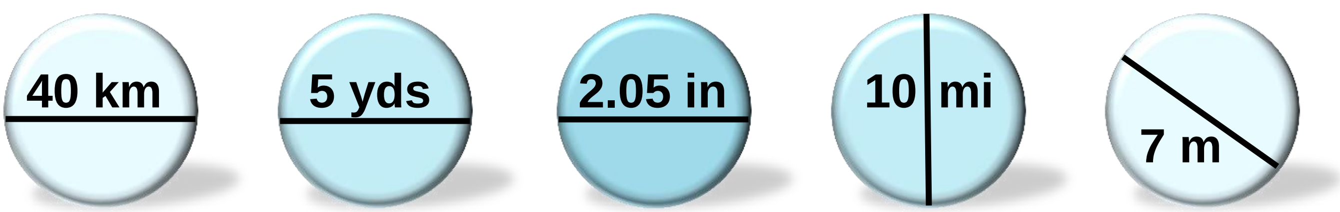
Find the diameter:



6. ________ 7. ________ 8. ________ 9. ________ 10. ________

11. Write the formula to find the circumference of a circle when given the diameter.

Use this formula to find the circumference. Use 3.14 for π.



12. ________ 13. ________ 14. ________ 15. ________ 16. ________

17. Write the formula to find the circumference of a circle when given the radius.

Use this formula to find the circumference. Express your answer in terms of π.

18. ________ 19. ________ 20. ________ 21. ________ 22. ________

The instructions for this portion say “answer to the nearest hundredth.” Where are the hundredths located, and how do we round to them? Consider these two numbers.

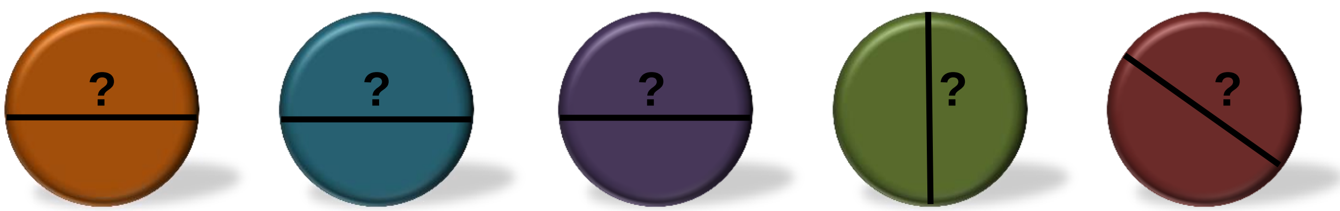
458.2736 **9,003.61682**

The hundredths place is the second digit after the decimal point. To decide which number to write there, we look at the third place after the decimal (the thousandths place).

- If the third digit is 4 or less, we keep the digit in the hundredths place. In 458.2736, the digit in the thousandths place -- 3 -- is 4 or less, so the answer is 458.27.
- If the third digit is 5 or more, we round up the digit in the hundredths place. In 9,003.61582, the digit in the thousandths place -- 6 -- is 5 or more, so the answer becomes 9,003.62.

23. Write the formula to find the diameter of a circle when given the circumference.

Use this formula to find the diameter. Use 3.14 for π. Answer to the nearest hundredth.



24. C = 30 cm 25. C = 14 in 26. C = 3.5 mi 27. C = 65 yds 28. C = 0.6 m

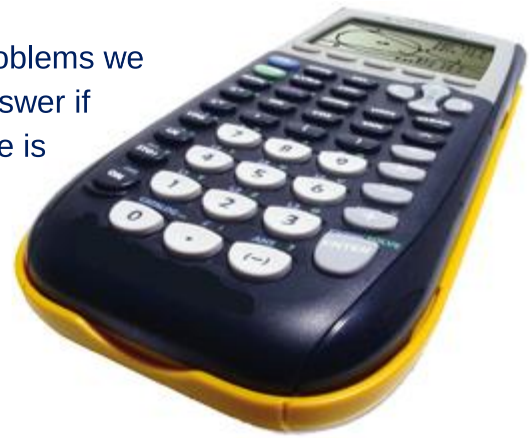
d = _________ d = _________ d = _________ d = _________ d = _________

29. Write the formula to find the radius of a circle when given the circumference.

Use this formula to find the radius. Use 3.14 for π. Answer to the nearest hundredth.

30. C = 2.9 ft 31. C = 105 m 32. C = 88 km 33. C = 11 yds 34. C = 7 mi

r = _________ r = _________ r = _________ r = _________ r = _________



Calculators are incredibly helpful with the types of problems we are doing. However, they only give us the correct answer if we enter the information properly. For example, there is a good chance that you may have answered 30 – 34 above incorrectly.

Consider: $$\frac{24}{(2)(3)} = r$$

To make things easy, we used 3 for pi in the example above.
This problem would actually be easier to do in our head than on the calculator. In the denominator we multiply 2 times 3 to get 6. 24 divided by 6 equals 4, so our radius is 4, and of course we write the units in our answer.

Now try it on the calculator. Did you get 4, or did you get 36? You will get the incorrect answer 36 if you key in the problem like this: 24 ÷ 2 x 3 =

To get the correct answer, 4, use the parentheses when keying in your equation:
24 ÷ (2 x 3) =

Check your answers for problems 30 – 34 above. If you found 4.55 ft for #30, re-do them. If your answer was 0.46 ft, then you keyed in the problem correctly. Remember, the circumference of a circle is a little more than three times as long as its diameter, and a little more than six times as long as its radius, so use estimation to check your answer.

Despite the wisdom of the t-shirt at left, pie *are* squared, at least for the purpose of remembering the formula for the area of a circle.

$$A = \pi r^2$$

Because the formula for the area of a circle only includes the radius of a circle, we have to calculate the radius if we are given the diameter. Simply divide the diameter by 2 to find the radius. Then square the radius (multiply it by itself) and multiply it by pi to find the area.

$$A = \pi\left(\frac{d}{2}\right)^2$$

Given the radius, find r^2.

35. r = 1,200 in 36. r = 23 cm 37. r = 19.2 ft 38. r = 0.3 mi 39. r = 7 m

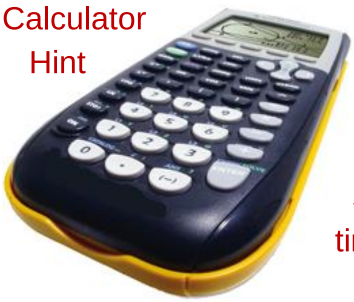
r^2 = ________ r^2 = ________ r^2 = ________ r^2 = ________ r^2 = ______

Given the diameter, find r^2.

40. *d* = 120 in 41. *d* = 23 cm 42. *d* = 19.2 ft 43. *d* = 0.3 mi 44. *d* = 7 m

r^2 = ________ r^2 = ________ r^2 = ________ r^2 = ________ r^2 = ______

Calculator Hint



When calculating the area of a circle, square the radius before multiplying by pi. In fact, if the formula had been written *after* the invention of the calculator (instead of thousands of years before), it would be $A = r^2\pi$. The answer is the same, but if you square the radius first, you will avoid the error of multiplying pi times the radius and then squaring both of those numbers.

Let's use these keys to calculate area: enter value of r, then [x^2] [x] [π] [=]

… and when given diameter: enter d, then [÷] [2] [=] [x^2] [x] [π] [=]

Given the radius, find the area of the circle to the nearest hundredth. Use 3.14 for π.

45. r = 4 mi 46. r = 5 cm 47. r = 5.2 km 48. r = 17 ft 49. r = 9.1 in

A = ________ *A* = ________ *A* = ________ *A* = ________ *A* = ________

50. Write the formula used to find the area of a circle when given its diameter.

Given the diameter, find the area of the circle to the nearest hundredth. Use 3.14 for π.

51. d = 24 ft 52. d = 6.4 cm 53. d = 20 mi 54. d = 6 yds 55. d = 43 in

A = ________ *A* = ________ *A* = ________ *A* = ________ *A* = ________

The base of a farmer's circular silo is 3,600 ft^2. What is the diameter or this silo?

We start with the formula for the area of a circle: $A = \pi r^2$

To find the diameter, we will ultimately have to double the radius, so let's isolate the *r* term.

If we divide both sides of the equation by pi, we will have this new representation of the area equation:

$$\frac{A}{\pi} = r^2$$

We know the area of the circular base of the silo, and pi never changes, so divide:

$3{,}600 \div 3.14 = r^2$

To increase accuracy, leave the quotient in your calculator instead of rounding the result.

$1{,}146.49681528 = r^2$

The inverse, or opposite, or squaring a number is finding its square root. Find the square root of both sides of the equation.

$\sqrt{1{,}146.49681528} = \sqrt{r^2}$

$\sqrt{r^2}$ is the same thing as *r*.

$33.86 = r$

Double the result to find the diameter.

$\underline{\text{x}2 \qquad \text{x}2}$

$67.17 = d$

d = 67.17 ft

Remember to include the units in the answer.
For area, the units are expressed as units squared.
The units for radius and diameter are not squared.

From this problem, we devised two new formulas we can use when given the area:

$$\sqrt{\frac{A}{\pi}} = r \qquad\qquad 2\left(\sqrt{\frac{A}{\pi}}\right) = d$$

Remember, divide by the area by pi before finding the square root.

Given the area of the circle, find the radius. Round to the nearest hundredth. Use 3.14 for π.

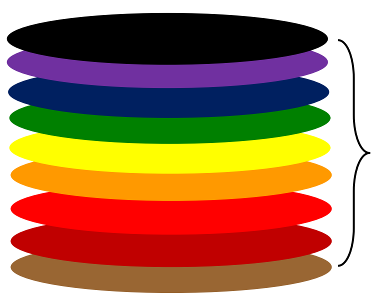
800 in^2	1,250 yds^2	390 km^2	2,408 ft^2	11,162 cm^2
56. *r* = ______	57. *r* = ______	58. *r* = ______	59. *r* = ______	60. *r* = _____

Given the circle's area, find the diameter. Round to the nearest hundredth. Use 3.14 for π.

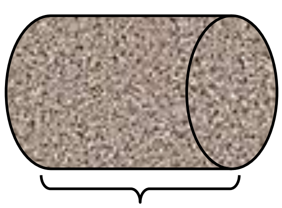
484 mm^2	999.8 ft^2	86,000 in^2	0.55 mi^2	8.612 km^2
61. d = ______	62. d = ______	63. d = ______	64. d = ______	65. d = _____

Cylinders

Cylinders are everywhere around us, from the Mountain Dew can to the farmer's silo, and most often serve as containers. The amount of drink, food, or other items that we store in these cylinders is called volume, and is easy to calculate. Now that we are somewhat expert at finding the area of a circle, we can imagine stacking an infinite number of circles on top of each other to attain a certain height. The area of the bottom circle, times the height of the cylinder equals the volume of the cylinder. Since a cylinder is a three-dimensional figure, the units of measurement associated with it have an exponent of three.



$$V = \underbrace{\pi r^2}_{\text{Area of a circle}} h$$



Does turning a cylinder on its side change its height? No, of course not. So we will define the height of a cylinder as the distance between its circular ends. The volume is found by multiplying the area of one circular end by the height.

Given the area of the base and the height, find the volume of the cylinders.

#66: 240 in^2, 15 in

#67: A = 486 in^2, h = 4.7 in

#68: h = 11 cm, ← A = 12 cm^2

66. V = __________

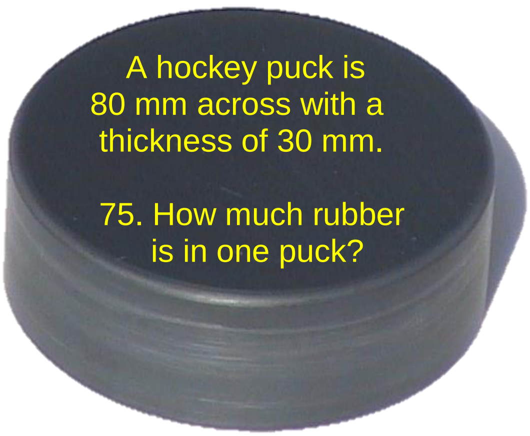
67. V = __________

68. V = __________

69. What is the formula for the volume of a cylinder?

Use this formula to find the volume of cylinders with the given radius and height.

70.	71.	72.	73.	74.
r = 15 ft	r = 63 cm	r = 150 mm	r = 4.89 in	r = 3 m
h = 80 ft	h = 12 cm	h = 420 mm	h = 9.62 in	h = 7 m
V = ____	V = ____	V = ____	V = ____	V = ____



A hockey puck is 80 mm across with a thickness of 30 mm.

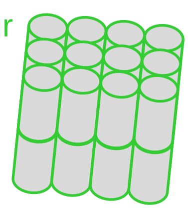
75. How much rubber is in one puck?

Spoiler Alert!

When you take Calculus in high school or college, you will be asked to find the most efficient size of a can. The companies that produce millions of cans of food each year (like Campbell's Soup) want to use the least amount of metal possible to produce each can. It turns out that the most efficient size is a can whose height is exactly twice as tall as its diameter.

76. How much soup fits in the most efficient-sized soup can if it is 4.6 inches tall?

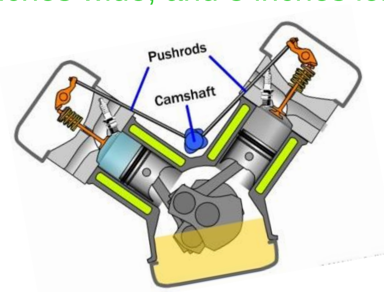
77. Suppose that the Dew in the cans stacked in this package (←) came in perfect cylinders, as opposed to the cans with the rounded tops and bottoms. Now suppose the cans were arranged in three rows of four and stacked two high, like this:

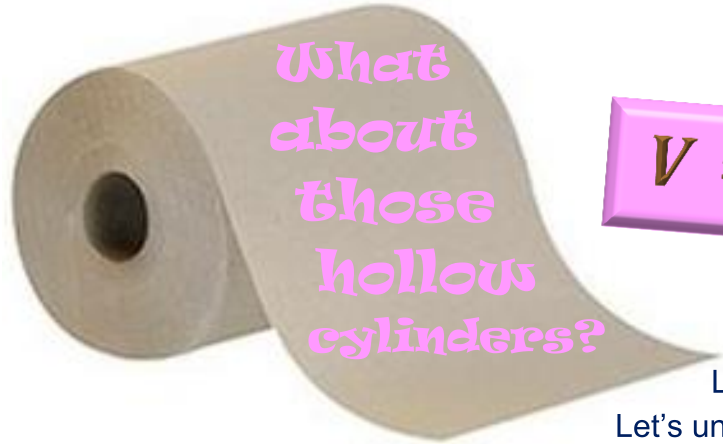


…and that the cans were filled to the very top with soda.

Find the total volume of soda inside the 24 cans if the box is 10 inches tall, 6 inches wide, and 8 inches long.

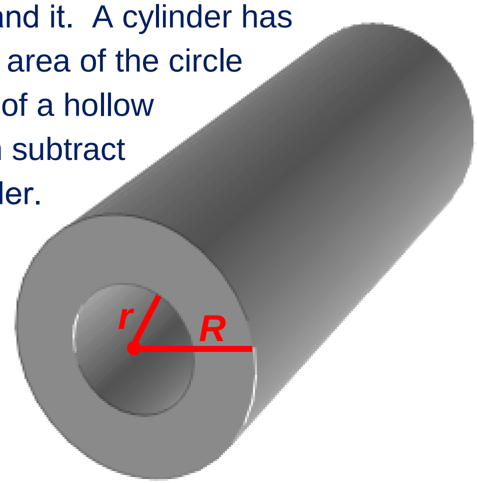
78. The 454 horsepower (volume) in a Chevy big-block engine is produced by 8 pistons which each have a 4-inch stroke (height). What is the diameter of these pistons?





$$V = \pi R^2 h - \pi r^2 h$$

Let's not get intimidated by this formula. Let's understand it. A cylinder has a circular base so we find its volume by multiplying the area of the circle times the height. Easy right? Now, to find the volume of a hollow cylinder, we find the volume of the whole cylinder, then subtract the volume of the hole in the middle – a "thinner" cylinder. As for the complicated looking formula, the ***R*** is the radius of the outer cylinder and the ***r*** is the radius of the inner cylinder. Everything else remains the same.



A paper towel roll stands one foot tall. It measures five inches across and the paper towels are wrapped around a cardboard tube which measure one inch across.

79. Write the formula to find the volume of a hollow cylinder.

80. List the value of *R*.

81. List the value of *r*.

82. List the value of *h*.

83. Find the volume of the paper towels themselves.

Farmer Joseph learned today how to calculate the volume of a cylinder. He used this knowledge to calculate the amount of water in his well and the quantity of corn in his tallest silo.

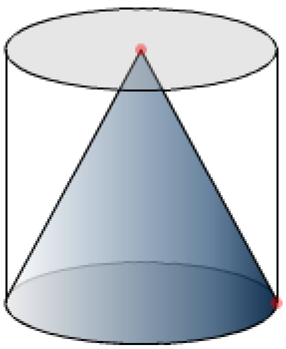
Use pi = 3.14, find to the nearest hundredth

84. Find the volume of the well, which is 48" across and has a depth of 31' 4" of water.

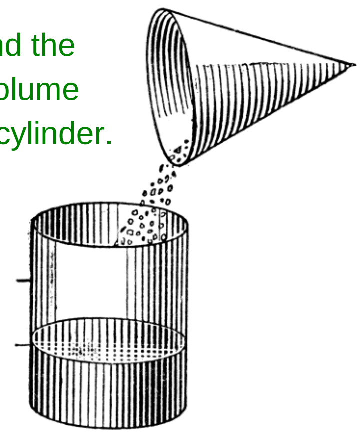
85. …and the silo, with *r* = 11' and *h* = 56' 3".

Cones

Cones are *not* all around us, except the sugar cones in ice cream stores -- and the orange cones at highway construction sites. They are not nearly as common as cylinders, but they do have an interesting relationship to cylinders. Take a look at these pictures.



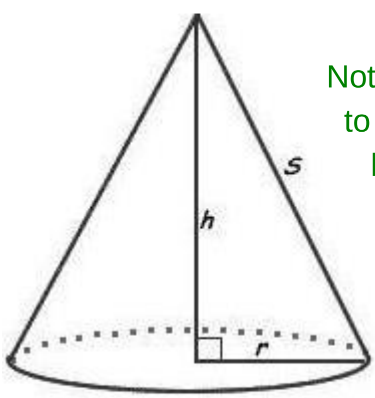
These two figures share the same base and the same height. When this is the case, the volume of the cone is exactly one-third that of the cylinder.



$$V = \frac{1}{3}\pi r^2 h$$

In fact, the volume of a cone is always one-third that of a cylinder with the same dimensions.

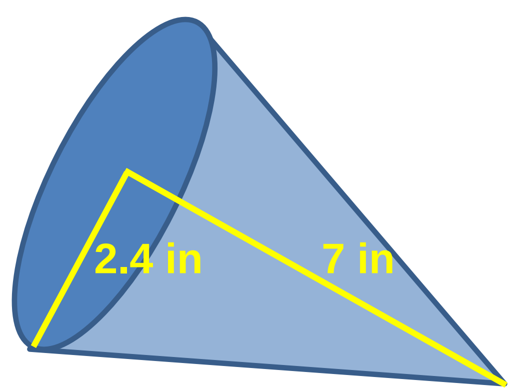
To key this into the calculator, find r^2, multiply it by h and pi, then divide the total by 3.



Notice, the height ***h*** is measured from the vertex (point) of the cone to the center of the base. The ***s*** is called the slant height. The slant height is used to find the surface area of a cone, but we do not use it to find the volume. The ***s*** is often included in volume problems to confuse you. Ignore it!

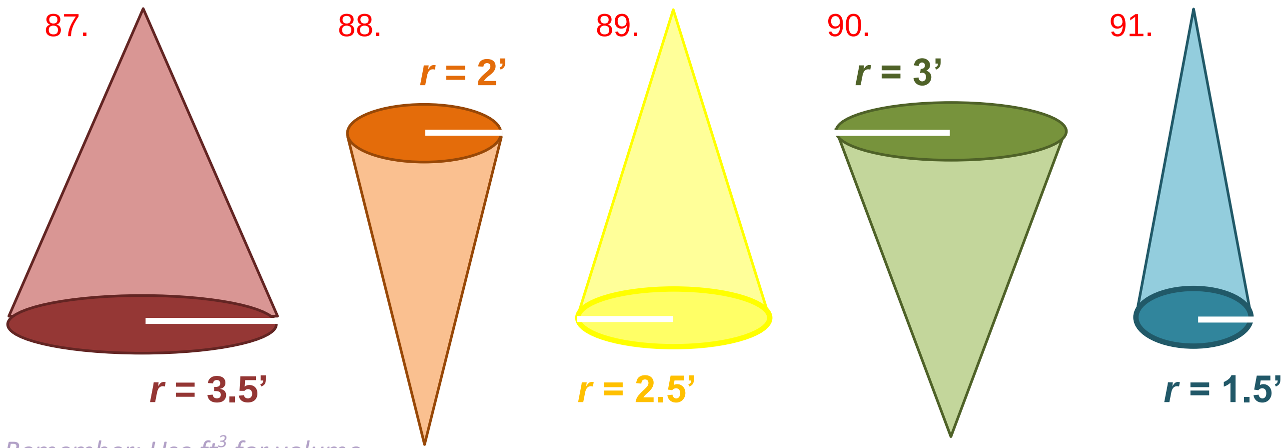
$$V = \frac{1}{3}\pi(2.4^2)7$$

$$V = 42.33\text{in}^3$$



86. Write the equation to find the volume of a cone.

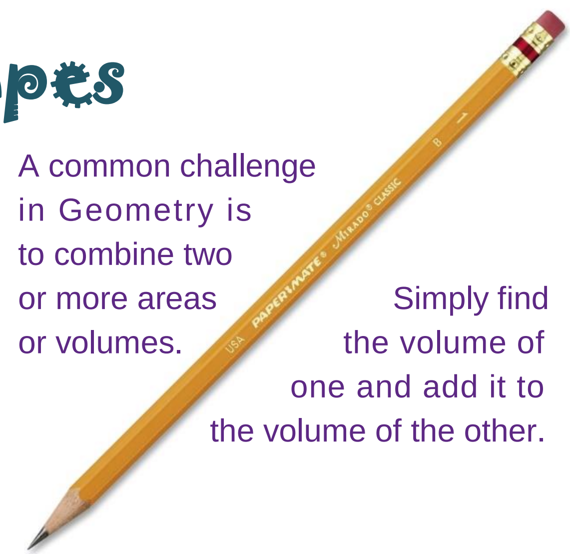
Find the volume of these cones, each of which have a **height of 8 feet**.



Remember: Use ft^3 for volume

V = ___________ V = ___________ V = ___________ V = ___________ V = _________

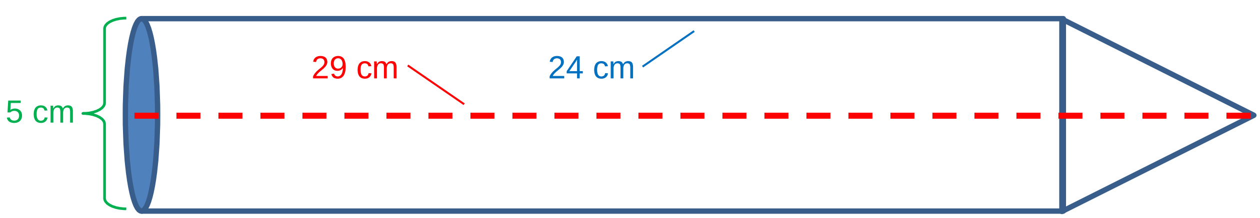
Let's Combine Shapes



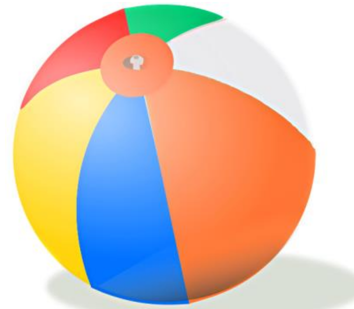
A common challenge in Geometry is to combine two or more areas or volumes.

Simply find the volume of one and add it to the volume of the other.

92. Find the volume of the figure below, which combines a cylinder with a cone. Use 3.14 for pi and write your answer to the nearest hundredth.



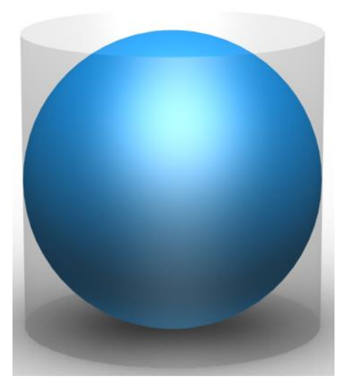
Spheres



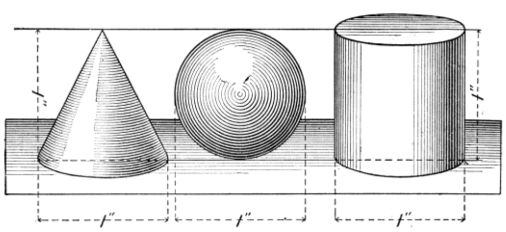
Spheres make LeBron James one of the most famous people on the planet (which also happens to be a sphere). I have always been amazed at the simplicity of the formula for the volume of a sphere. Pi is a very strange number -- an irrational number that goes on forever without ever repeating. But the volume of a sphere combines pi with a very simple fraction, four-thirds.

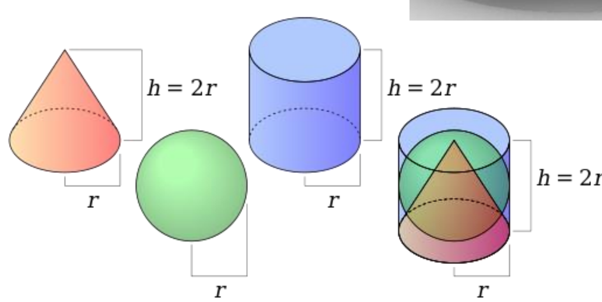
$$V = \frac{4}{3}\pi r^3$$

Q. A cone that fits inside a cylinder fills up exactly one-third of the cylinder. Is there a similar relationship between a sphere and a cylinder?



A. Yes. A sphere that fits exactly inside a cylinder takes up two-thirds of the space. So if we combine the three figures, the cone takes up 1/3 of the cylinder and the sphere takes up the other 2/3's of the volume.





Consider: The volume of a sphere is $\frac{4}{3}\pi r^3$. The volume of a cylinder is $\pi r^2 h$. When a sphere fits exactly inside a cylinder, they have the same radius and their height is $2r$.

Compare: sphere → $\frac{4}{3}\pi r^3$ cylinder → $\pi r^2 h = \pi r^2 2r = \pi r^3 2$

Divide each formula by πr^3 and this is what remains: sphere → $\frac{4}{3}$ cylinder → 2

Simplify the ratio → $\frac{4}{3} : 2 = \frac{4}{3} : \frac{6}{3} = 4:6 = 2:3$ Therefore, the sphere is 2/3 as big as the cylinder.

$$V = \frac{4}{3}\pi r^3$$

The formula is not that hard to memorize, but how do we key it into a calculator? As always, I recommend entering the radius first, so that when you cube it (raise it to the third power), you don't accidentally cube all the other numbers you entered. Use the carat key to raise a number to the third power: ^3. Also, to multiply a number by 4/3, you multiply it by 4, then divide it by 3.

Let's find the volume of a rubber ball that has a diameter of 10 inches.

A diameter of 10, means the radius is 5. Replace *r* with 5, and use the π key.

$$V = \frac{4}{3}\pi(5)^3$$

5	^	3	x	π	x	4	÷	3	=

Of course there are many different ways to enter the same information.

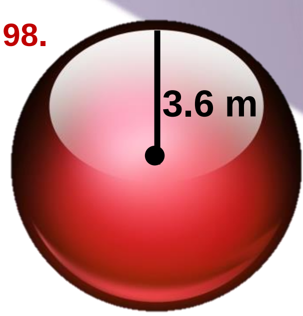
Cube the given radius of each sphere.

93. r = 2.6 cm   94. r = 9 ft   95. r = 11.6 m   96. r = 10 in   97. r = 68 mm

r^3 = _____ r^3 = _____ r^3 = _____ r^3 = _____ r^3 = ______

Given the radius of each sphere, find its volume.

98.



99.

Given the diameter, find the volume.

100.