Georgia
Standards of Excellence
Curriculum Frameworks

Mathematics

GSE Grade 8
Unit 7: Solving Systems of Equations

Georgia Department of Education

Richard Woods, Georgia's School Superintendent
"Educating Georgia's Future"

Unit 7
Solving Systems of Equations

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OVERVIEW

In this unit students will:

- understand the solution to a system of equations is the point of intersection when the equations are graphed;
- understand the solution to a system of equations contains the values that satisfy both equations;
- find the solution to a system of equations algebraically;
- estimate the solution for a system of equations by graphing;
- understand that parallel lines have will have the same slope but never intersect; therefore, have no solution;
- understand the two lines that are co-linear share all of the same points; therefore, they have infinitely many solutions; and
- apply knowledge of systems of equations to real-world situations.

This unit extends solving equations to understanding solving systems of equations, or a set of two or more linear equations that contain one or both of the same two variables. Once again the focus is on a solution to the system. Student experiences are with numerical and graphical representations of solutions. Beginning work involves systems of equations with solutions that are ordered pairs of integers, making it easier to locate the point of intersection, simplify the computation, and hone in on finding a solution. More complex systems are investigated and solved by using graphing technology.

Contextual situations relevant to eighth graders add meaning to the solution to a system of equations. Students explore many problems for which they must write and graph pairs of equations leading to the generalization that finding one point of intersection is the single solution to the system of equations. Students connect the solution to a system of equations, by graphing, using a table, and writing an equation. Students compare equations and systems of equations, investigate using graphing calculators or graphing utilities, explain differences verbally and in writing, and use models such as equation balances.

Problems are structured so that students also experience equations that represent parallel lines and equations that are equivalent. This will help them to begin to understand the relationships between different pairs of equations. When the slope of the two lines is the same, the equations are either different equations representing the same line (thus resulting in many solutions), or the equations are different equations representing two non-intersecting, parallel, lines that do not have common solutions.

System-solving in Grade 8 includes estimating solutions graphically, solving using substitution, and solving using elimination. Students gain experience by developing conceptual skills using models that develop into abstract skills of formal solving of equations. Students also have to change forms of equations (from a given form to slope-intercept form) in order to compare equations.
Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. To assure that this unit is taught with the

appropriate emphasis, depth, and rigor, it is important that the tasks listed under "Evidence of Learning" be reviewed early in the planning process. A variety of resources should be utilized to supplement this unit. This unit provides much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the types of learning activities that should be utilized from a variety of sources.

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

STANDARDS FOR MATHEMATICAL CONTENT

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

BIG IDEAS

- There are situations that require two or more equations to be satisfied simultaneously.
- There are several methods for solving systems of equations.
- Solutions to systems can be interpreted algebraically, geometrically, and in terms of problem contexts.
- The number of solutions to a system of equations can vary from no solution to an infinite number of solutions.

ESSENTIAL QUESTIONS

- What does the point of intersection mean?
- What is a system of equations?
- What does it mean to solve a system of linear equations?
- How do I decide which method would be easier to use to solve a particular system of equations?
- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?
- How can I interpret the meaning of a "system of equations" algebraically and geometrically?
- What does the geometrical solution of a system mean?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Identify and calculate slope
- Identify y-intercept
- Create graphs given data
- Analyze graphs
- Make predictions given a graph

FLUENCY

It is expected that students will continue to develop and practice strategies to build their capacity to become fluent in mathematics and mathematics computation. The eventual goal is automaticity with math facts. This automaticity is built within each student through strategy development and practice. The following section is presented in order to develop a common understanding of the ideas and terminology regarding fluency and automaticity in mathematics:

Fluency: Procedural fluency is defined as skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Fluent problem solving does not necessarily mean solving problems within a certain time limit, though there are reasonable limits on how long computation should take. Fluency is based on a deep understanding of quantity and number.

Deep Understanding: Teachers teach more than simply "how to get the answer" and instead support students' ability to access concepts from a number of perspectives. Therefore students are able to see math as more than a set of mnemonics or discrete procedures. Students demonstrate deep conceptual understanding of foundational mathematics concepts by applying them to new situations, as well as writing and speaking about their understanding.
Memorization: The rapid recall of arithmetic facts or mathematical procedures. Memorization is often confused with fluency. Fluency implies a much richer kind of mathematical knowledge and experience.
Number Sense: Students consider the context of a problem, look at the numbers in a problem, make a decision about which strategy would be most efficient in each particular problem. Number sense is not a deep understanding of a single strategy, but rather the ability to think flexibly between a variety of strategies in context.

Fluent students:

- flexibly use a combination of deep understanding, number sense, and memorization.
- are fluent in the necessary baseline functions in mathematics so that they are able to spend their thinking and processing time unpacking problems and making meaning from them.
- are able to articulate their reasoning.
- find solutions through a number of different paths.

For more about fluency, see: http://www.youcubed.org/wp-content/uploads/2015/03/FluencyWithoutFear-2015.pdf and: http://joboaler.com/timed-tests-and-the-development-of-math-anxiety/

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary. Note – At the elementary level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 7 mathematics.

Visit http://intermath.coe.uga.edu or http://mathworld.wolfram.com to see additional definitions and specific examples of many terms and symbols used in grade 8 mathematics.

- **System of Linear Equations:**
- **Simultaneous Equations:**

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Guide.

SPOTLIGHT TASKS

A Spotlight Task has been added to each MGSE mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

TASKS

Task Name	Task Type/Grouping Strategy	Content Addressed	Standard(s)
Walk the Plank – Action Figure Edition	Learning Task *Partner/Small Group*	Explore systems of equations to make sense of slope, intersections of lines and y-intercept within a context	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8c
Playing Catch Up (Spotlight Task)	Learning Task *Partner/Small Group*	Exploring systems of equations to solve a real-world problem.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Cara's Candles	Performance Task *Individual/Partner*	Use of multiple representations to demonstrate their understanding of a system of linear functions.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
DVD Club	Performance Task *Partner/Small Group*	Use of multiple representations to demonstrate their understanding of a system of linear functions.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Stacking Cups (Spotlight Task)	Learning Task *Partner/Small Group*	Exploring systems of equations to solve a real-world problem.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Field Day	Performance Task *Individual/Partner*	Use a system of linear functions to solve a problem situation.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Free Throw Percentages	Learning Task *Partner/Small Group*	Use a system of linear functions to solve a problem situation.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
How Much Did They Cost?	Performance Task *Individual/Partner*	Use a system of linear functions to solve a problem situation.	MGSE8EE.8 MGSE8EE.8b MGSE8EE.8c
Fencing	Short Cycle Task *Individual/Partner*	In this task, you must figure out the cost of building fences from fence posts and wooden panels.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Playing with Straws	Performance Task *Individual/Partner*	Use a system of linear functions to solve a problem situation.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Planning a Party	Performance Task *Individual/Partner*	Use a system of linear functions to solve a problem situation.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c

Coefficents and Contant Ratios. . . They Mean Something?	Learning Task Partner/Small Group	Look for patterns in systems of equations to develop efficient strategies for working with them.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
What Are the Coefficients?	Performance Task *Individual/Partner*	Use a system of linear functions to solve a problem situation.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Cell Phone Plans	Performance Task *Individual/Partner*	Use a system of linear functions to solve a problem situation.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Optimization Boomerang	Formative Assessment Lesson *Partner/Small Group*	Use system of equations to find the maximize amount of money which could be raised.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Classifying Solutions to Systems of Equations	Formative Assessment Lesson *Partner/Small Group*	Classify solutions to a pair of linear equations by considering their graphical representations.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Stained Glass Window	Culminating Task *Individual/Partner*	Graph linear equations and explain the points of intersection.	MGSE8EE.8 MGSE8EE.8a MGSE8EE.8b MGSE8EE.8c
Technology Resources			

Walk the Plank – Action Figures Edition

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Adapted from the illuminations lesson: Walk the Plank http://illuminations.nctm.org/Lesson.aspx?id=2347

STANDARDS FOR MATHEMATICAL CONTENT

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MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

ESSENTIAL QUESTIONS

- What does the point of intersection mean?
- What is a system of equations?
- What does it mean to solve a system of linear equations?

MATERIALS NEEDED

- Kitchen Scale (1 per small group of students)
- A plank (a meter stick will work)
- A book that has about the same height as the scale.
- Some action figures (be sure they can stand on their own) If you don't have any at your house, your students will.
- Markers, crayons or colored pencils
- Graph paper or Desmos
- Task review questions (attached)

TEACHER NOTES:

This task is adapted from the Illuminations task Walk the Plank. It has been adapted to take students' weight completely out, yet engage all students in collecting data and stay true to the content investigated in the original task.

The task involves placing one end of a wooden "plank" on a kitchen scale and the other end on a thick book. Students can choose two or three action figures to "walk the plank" and record the weight measurement as the distance of the action figure from the scale changes. The results are unexpected— the relationship between the weight and distance is linear, and all lines meet at the same point, regardless of the weight of the action figure. This investigation engages students in modeling a "real world" occurrence of negative slope. Examples of negative slopes in context can be hard to find.

Prior to beginning the task, students need to calibrate the scale (make sure it is set to zero, before beginning).

COMMON MISCONCEPTIONS:

- *Students may think that there is always one solution to systems of equations. This misconception should be addressed, as many of the others are, through contexts and contextual problems and sense-making. The contexts used in the problems we give students should engage students in reasoning and lead them to conclude when a system has no solutions, multiple solutions, or just one solution.*
- *Students may interpret the constraints and variables incorrectly. This can be addressed by focusing students on each of the constraints and/or meanings of the variables individually. Student discussions of these constraints before diving into the task may also be helpful.*

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Once students have calibrated their scales, it's important that they set up the plank properly. The plank should lay across the kitchen scale so that the 2cm or 3cm mark on the meter stick is in the center of the scale. This mark on the meter stick will be where students begin measuring. It is recommended that teachers and students discuss what they are about to do in order to build an understanding of what numbers they will be seeing on the meter stick and what numbers they will collect for their data. For example, the number at the center of the scale may be 2 cm, but how far is it from the scale? *(zero cm)* Likewise, the meter stick should extend onto the book for at least 4 cm.

As soon as students have set up, allow them to choose at least two action figures, a smaller one and a larger one. Just a side note, I recommend that you do not use Barbie dolls for this task. Their feet will not allow them to stand on their own very well.
Students should choose one to begin with and make them "Walk the Plank." Students place their first action figure on the plank (meter stick) at the center of the scale, write down the distance from the scale (0), and the weight (combined weight of the plank and the action figure).
Students can choose an interval for walking the plank or you may assign one to them (anywhere from 5 to 10 cm seems to work well.

Students collect the data for distance from the scale and weight for both action figures. Using a table works nicely here. After the data is collected, students should plot the points on a graph (using a different color for each action figure's data). This can be done with graph paper and colored pencils or by using a graphing tool such as Desmos. Once the data is graphed, students should draw a line of best fit (this can also be done with Desmos).

Once all data is graphed, students should complete the task review questions (attached).

Review student responses as a whole group. The goal here is to build a common understanding of the properties of some systems of equations, to interpret the intersection, to explain why the points are where they are. Students should be making sense of the mathematics through the context of this investigation.

DIFFERENTIATION:

Extension:

- Ask students to write a prediction about what would happen if a really heavy action figure was graphed. Give students a heavy object (it doesn't have to be an action figure – a 1 liter bottle of soda will work nicely) and have them test their theory and write about their findings to share with the class.

Intervention/Scaffolding:

- Struggling students may need go back to the investigation set up and place each action figure (separately) on the plank at the graph's intersection point. This can help students make sense of the graph and help them make sense of the mathematics of the problem (the numbers of the points on the graph explain the context of the action figures "walking the plank").

<u>Walk the Plank – Action Figures Edition</u> **Task Review** (Sample)

1. Record the data you collect for each of the action figures you chose in the tables below.

Leonardo

Distance	weight
0	7.8
4	7.6
8	7.4
12	7.2
16	6.8
20	6.5
24	4.6

44.4.6

Big Guy

0	10.8
4	10.5
8	10
12	9.5
16	9.0
20	8.4
24	7.8
44	4.7

What do you notice?

At the end they were close to the same weight. Big Guy was going by 5's at first.

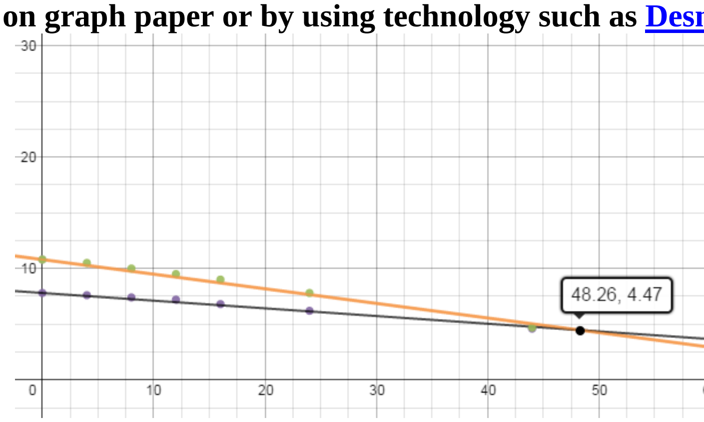
What do you wonder?

-How are they close to the same weight at the end.

(At the end, they were close to the same Weight. Big Guy was going by 5's at first.)

(-How are they close to the same weight at the end)

2. Graph this data on graph paper or by using technology such as [Desmos](https://www.desmos.com).



What do you notice? **What do you wonder?**

3. Draw a line of best fit on your graph for each set of points or use technology such as Desmos.

What does the slope represent in the context of this situation?

The slope represents the rate at which the weight of the action figure decreases as its distance from the scale increases.

What does the y-intercept represent in the context of this situation?

The y-intercept represents the combined weight of the plank and the action figure.

Using the lines of best fit, write equations for each of the lines representing each of the action figures.

Answers will vary.

4.  **Imagine that a 64 ounce Kool-Aid Man walked the plank.**

Answers will vary.

What weight would be shown on the scale if the Kool-Aid Man stood at the half-way mark?

Answers will vary.

Where would the Kool-Aid Man need to stand for the scale to show a weight of 32 ounces?

Answers will vary.

Walk the Plank – Action Figures Edition Task Review

1. Record the data you collect for each of the action figures you chose in the tables below.

What do you notice?

What do you wonder?

2. Graph this data on graph paper or by using technology such as Desmos.

What do you notice?

What do you wonder?

3. Draw a line of best fit on your graph for each set of points or use technology such as Desmos.

What does the slope represent in the context of this situation?

What does the y-intercept represent in the context of this situation?

Write an equation for the line representing the weights shown when the first action figure walked the plank, and write an equation for your line.

4.  **Imagine that a 64 ounce Kool-Aid Man walked the plank.**

What weight would be shown on the scale if the Kool-Aid Man stood at the half-way mark?

Where would the Kool-Aid Man need to stand for the scale to show a weight of 32 ounces?

Playing Catch Up (Spotlight Task)

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Task adapted from http://threeacts.mrmeyer.com/playingcatchup/ authored by Dan Meyer.

STANDARDS FOR MATHEMATICAL CONTENT

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

ESSENTIAL QUESTIONS

- How can systems of equations be used to solve real-world problems?

MATERIALS NEEDED

- **3-Act Student Answer Sheet.**
- **Video 1:** http://threeacts.mrmeyer.com/playingcatchup/act1/act1.mov
- **Video 2:** http://threeacts.mrmeyer.com/playingcatchup/act3/act3.mov
- **Image 1:** http://threeacts.mrmeyer.com/playingcatchup/act2/juliojones-time.png
- **Image 2:** http://threeacts.mrmeyer.com/playingcatchup/act2/richeisen-time.png

TEACHER NOTES

In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

TASK DESCRIPTION

The following 3-Act Task can be found at: http://threeacts.mrmeyer.com/playingcatchup/

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:

Watch the video: (link: http://threeacts.mrmeyer.com/playingcatchup/act1/act1.mov)
Ask students what questions they have about the video. Suggestions:

1. Who will win?
2. Write a guess.

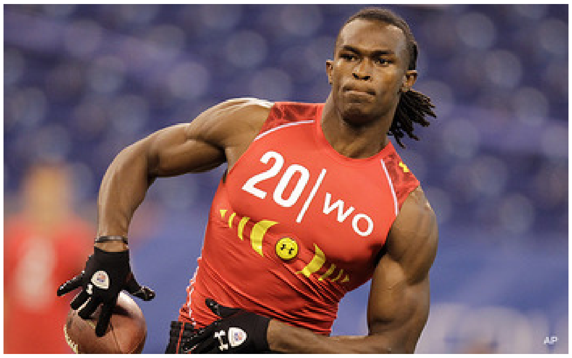
ACT 2:

What information would be useful to know here?
Image 1: Julio Jones statistics, http://threeacts.mrmeyer.com/playingcatchup/act2/juliojones-time.png

NFL Combine 2011 Results: Julio Jones and 10 Pro Prospects Who Shined Sunday

By Ryan Braun (Analyst) on February 27, 2011

1. Julio Jones, WR, Alabama



Associated Press

Height: 6'2.75"

Weight: 220

40-yard dash: 4.39

Vertical Leap: 38.5

Broad Jump: 11'3"

Bench: 17

Julio Jones is kind of a beast and, by most accounts, he stole the day. The Alabama receiver was very sharp, explosive out of his breaks and demonstrated tremendous functional speed.

And then there were the running/jumping drills, in which Jones tested off the charts. The 4.39-second 40 is exceptional for a 220-pound man, and it is more then a 10th-of-a-second faster then consensus top-rated receiver A.J. Green.

It didn't stop there: Jones's broad jump was two inches below the combine's best since 2000, his vertical leap was, again, astonishing for a 220-pound man. And he looked like he does in the picture above.

Fluidity, speed, picture above.

Julio Jones wins Day 4 of the combine.

Image 2: Rich Eisen statistics, http://threeacts.mrmeyer.com/playingcatchup/act2/richeisen-time.png

ACT 3
Video: The answer, http://threeacts.mrmeyer.com/playingcatchup/act3/act3.mov

ACT 4- Sequel
What kind of head start would have allowed Julio Jones to beat Rich Eisen?

Playing Catch Up

Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?		
Place an estimate that is too high and too low on the number line		
Low estimate	*Place an "x" where your estimate belongs*	High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_____________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Cara's Candles

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

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MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.

TEACHER NOTES:

The context of this task is to help students connect systems of equations to a "real world" situation. Additionally, the questions Cara wonders about lead students to look at different points on the graph and determine how they are related to the context of the problem. This should be helpful to make these connections explicit.

COMMON MISCONCEPTIONS:

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

ESSENTIAL QUESTIONS:

- How can I interpret the meaning of a "system of equations" algebraically and geometrically?
- What does it mean to solve a system of linear equations?
- How can the solution to a system be interpreted geometrically?
- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?

MATERIALS:

- Colored pencils
- Straightedge
- Graphing calculator *(optional)*
- Graph paper http://incompetech.com/graphpaper
- Copies of task for students

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use multiple representations to demonstrate their understanding of a system of linear functions.

A classroom video of Georgia eighth grade students performing this task may be found at: http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathClub

DIFFERENTIATION:

Extension:

- Change the rates at which the candles burn from 2.5cm to 2.25cm and from 1.5cm to 1.75cm. This will result in students having to take one additional step to solve the problem. The table will have to be increased in order to accommodate 8 hours (the solution).

Intervention/Scaffolding:

- Encourage struggling students to use the table to graph the system and find the point of intersection as the solution.

Cara's Candles

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour which it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

Cara needs your help to determine whether these two candles would ever reach the same height at the same time if allowed to burn the same length of time. She also wants to know what height the two candles would be at that time. If it is not possible, she wants to know why it could not happen and what would need to be true in order for them to be able to reach the same height. To help Cara understand what you are doing, be sure to use multiple representations, justify your results, and explain your thinking.

Time (hours)	16 cm candle height (cm)	12 cm candle height (cm)
0		
1		
2		
3		
4		
5		
6		
7		

Solutions

Equations:

If x represents the number of hours that the candles burn and y represents the height of the candle after burning x hours, then the two equations that model this problem situation are

$$y = 16 - 2.5x \text{ and } y = 12 - 1.5x.$$

The system is easily solved by substitution, resulting in

Step 1:	$16 - 2.5x = 12 - 1.5x$
Step 2:	$-12 + 2.5x = -12 + 2.5x$
Step 3:	$4 = x \text{ or } x = 4$

Thus, the candles reach the same height after 4 hours. At that time both candles are 6 cm long, since

16 cm candle	*12 cm candle*
$16 - 2.5(4)$	$12 - 1.5(4)$
$16 - 10 = 6\ cm$	$12 - 6 = 6\ cm$

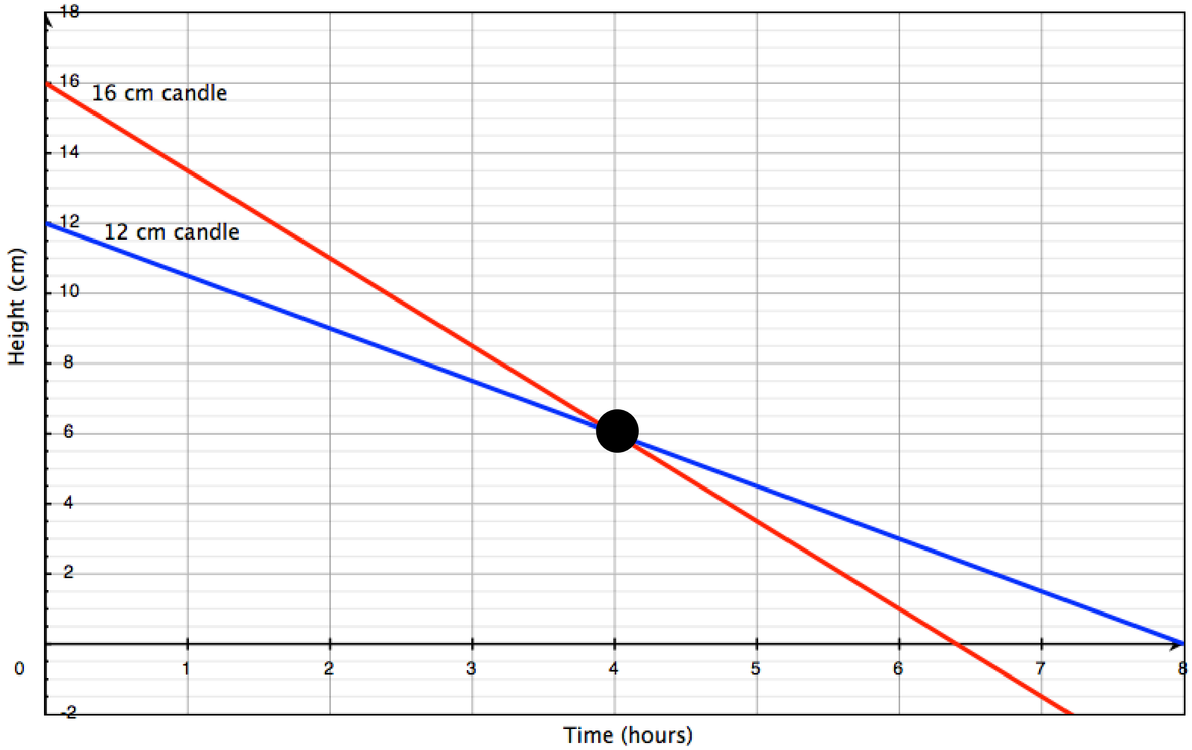
<u>Table:</u>

Supporting the work with a table encourages the student to take note of the changing values of the heights over time and think about the reasonableness of the results.

Time (hours)	*16 cm candle height (cm) 16 – 2.5x*	*12 cm candle height (cm) 12 – 1.5x*
0	*16*	*12*
1	*13.5*	*10.5*
2	*11*	*9*
3	*8.5*	*7.5*
4	*6*	*6*
5	*3.5*	*4.5*
6	*1*	*3*
7	*-1.5*	*1.5*

Use the opportunity to bring out the concept of the natural restrictions.

- ***For instance when x = 7 in the first function, the candle would have a negative height, which is impossible.***



A sample of Georgia eighth-grade student work is shown on the next page.

Task 1

Cara's Candles

$16 - 2.5x = 12 - 1.5x$

$4 = 1x$

$x = 4$

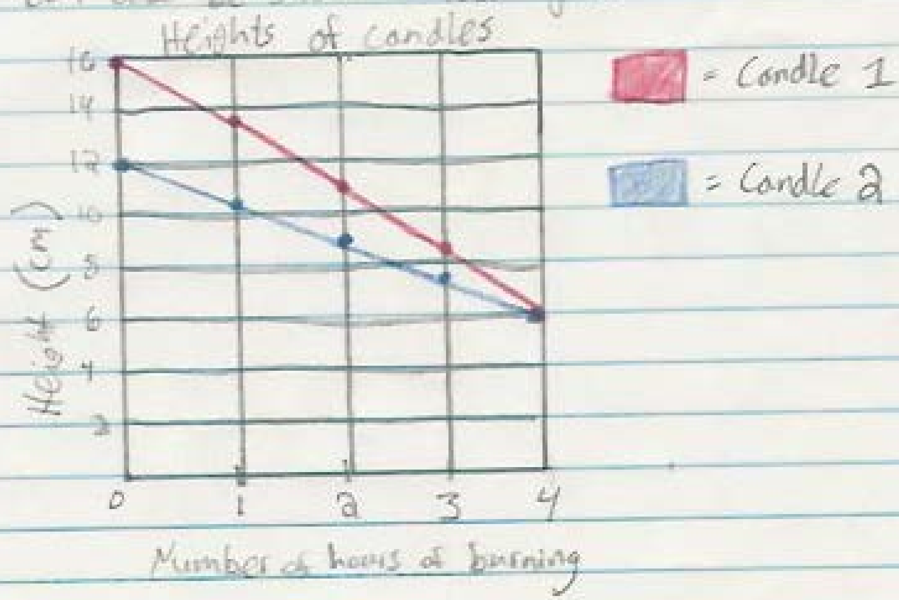
After four hours, the candles will be the same height. They will both be six centimeters tall.

The decrease in height can be shown in this table.

hours	height of candle 1	height of candle 2
0	16 cm	12 cm
1	13.5 cm	10.5 cm
2	11 cm	9 cm
3	8.5 cm	7.5 cm
4	6 cm	6 cm

-2.5 (candle 1 decreases each hour)   -1.5 (candle 2 decreases each hour)

It can also be shown in this graph



Teacher Commentary

The student accurately gives representations in the form of an equation, a table, and a graph. He clearly states the result in complete sentences. His graph is clearly labeled and uses appropriate scales. His work demonstrates that he can translate from a problem context to equations, that he comprehends the meaning of a point of intersection of two graphs, and that he can interpret that meaning in terms of the original problem situation.

Cara's Candles

Cara likes candles. She also likes mathematics and was thinking about using algebra to answer a question that she had about two of her candles. Her taller candle is 16 centimeters tall. Each hour which it burns makes the candle lose 2.5 centimeters in height. Her short candle is 12 centimeters tall and loses 1.5 centimeters in height for each hour that it burns.

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Time (hours)	16 cm candle height (cm)	12 cm candle height (cm)
0		
1		
2		
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7		

DVD Club

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHER NOTES:

In order for students to be successful, the following skills and concepts need to be maintained:

- MCC5.G.1, MCC5.G.2, MCC6.EE.5, MCC6.EE.6, MCC6.EE.7, MCC7.EE.4

COMMON MISCONCEPTIONS:

- *Students may infer that only x and y can be used for variables. This may become problematic when students are given equations with different variables. To address this misconception, discuss with students which variables they would like to use within a certain context. Groups of students may have different ideas about which variables to use, but the idea that any variable will work, as long as it is defined in the beginning, is the big idea here.*

- *Students may infer that the variables should always be on the left side of the equations. This can be addressed early on (and is even being addressed in the elementary grades where this misconception begins to grow) by making sure students consistently see equations with variables on either side.*
- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations. Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.*
- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

ESSENTIAL QUESTIONS:

- How can I interpret the meaning of a "system of equations" algebraically and geometrically?
- What does it mean to solve a system of linear equations?
- How can the solution to a system be interpreted geometrically?
- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?

MATERIALS:

- Colored pencils
- Straightedge
- Graphing calculator *(optional)*
- Graph paper http://incompetech.com/graphpaper
- Copies of task for students

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use multiple representations to demonstrate their understanding of a system of linear functions.

A classroom video of Georgia eighth grade students performing this task may be found at: http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathClub

DIFFERENTIATION:

Extension:

- Instruct students to come up with another situation similar to the situation presented in the DVD Club Task. Write algebraic expressions for the newly created situations and determine the solution that works for both equations as it relates to the context of the problem created.

Intervention/Scaffolding:

- Work through a similar example with struggling students prior to beginning this task. Be sure to facilitate students' understanding of the context before tackling this task.

DVD Club

A group of eighth-graders at your school is thinking of forming a club whose members rent DVDs. The club would also rent DVDs to non-members but at a higher price.

1. What do you think would be a reasonable amount students would be willing to pay for a year's membership in the club?

2. What do you think would be a fair price for a member to rent one DVD for one week?

3. What do you think would be a fair price for a non-member to rent one DVD for one week?

4. Using the amounts you decided on, write algebraic equations that would represent the total cost y for a member to rent x DVDs in one year and another algebraic equation that would represent the cost y for a non-member to rent x DVDs in one year.

5. How many DVDs would a person have to rent in one year in order for the cost to be the same whether one is a member or not? Show how you know.

Solution

Surveying students to determine what "prospective customers" consider reasonable charges could promote connections with statistics as well as the development of number sense. By allowing students to choose the numbers, the task creates a realistic problem situation and fosters their engagement. It also allows for the numbers chosen to not be contrived as they tend to be in textbook situations. Thus, the system which models the situation could have a solution that is not a whole number, requiring students to think more deeply about the meaning of the result.

One possible solution would be that students decide to charge $20 per year for membership, $4 per DVD for members to rent, and $5 per DVD for non-members to rent. Thus, $20 + 4x = 5x$ yields that the cost is the same if the customer rents 20 DVDs per year. Therefore, the customer would have to rent more than 20 in order to be paying less than a non-member.

To illustrate using another possible set of numbers, let membership be $25 per year, with costs of $3 per DVD for members and $5 per DVD for non-members. Then $25 + 3x = 5x$ yields the same cost per year when 12.5 DVDs are rented. Since it is impossible to rent ½ video, the conclusion would be that the cost is less with membership when 13 or more DVDs are rented.

Because it is logical to use substitution for solving this system, students may think of the problem as only involving one unknown. To encourage thinking about the situation as having two variables, also ask what the total cost is for the number of DVDs that results in the same cost whether the customer is a member or not.

A sample of Georgia eighth-grade student work is shown below.

DVD club

member – $y=2x+10$ to get this equation – y = member, 2 – price for one DVD, x – the number of DVDS, 10 the price for a membership.

non-member – $y=5x$ to get this equation y = non-member, 5 – price for one DVD, x – number of DVDS

To find the answer you have to find two points on a graph, 20,50 - member, 10,50 - non-member. Then you plug in the two points into their corresponding equations.

member - $50=2(20)+10$

non-member - $50=5(10)$

Therefore the number in place of the x in both equations is the number of DVDS that person can rent to pay the same amount as the other person. So a member can rent twenty DVDS for the same price as a non-members ten DVDS.

Teacher Commentary

The student clearly identifies the meaning of the variables and constants in the equations he has set up. His interpretation of the question led him to find two total prices that were the same and calculate two x values (numbers of DVDs) that yielded the same price. Geometrically his answer gives two points on the two lines that have the same y value rather than the point of intersection of the two lines. For his parameters, in order for the same number of DVDs to produce the same price,

5x = 2x + 10, which yields x = 3 1/3. Interpreting this result in the problem situation which would not permit renting 1/3 of a DVD, the student could conclude that belonging to the club results in a lower total price if four or more DVDs are rented.

DVD Club

A group of eighth-graders at your school is thinking of forming a club whose members rent DVDs. The club would also rent DVDs to non-members but at a higher price.

1. What do you think would be a reasonable amount students would be willing to pay for a year's membership in the club?

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5. How many DVDs would a person have to rent in one year in order for the cost to be the same whether one is a member or not? Show how you know.

<u>Stacking Cups</u> (Spotlight Task)

Back to Task Table

Task adapted from http://www.estimation180.com/stackingcups.html

STANDARDS FOR MATHEMATICAL CONTENT

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
6. Attend to precision.

ESSENTIAL QUESTIONS

· How can systems of equations be used to solve real-world problems?

MATERIALS NEEDED

- **3-Act Student Answer Sheet.**
- **Videos: http://www.estimation180.com/stackingcups.html**
- **Image 1:** http://www.estimation180.com/uploads/1/3/8/8/13883834/stacking_cups_act_2a.jpg
- **Image 2:** http://www.estimation180.com/uploads/1/3/8/8/13883834/stacking_cups_act_2b.jpg

TEACHER NOTES

In this task, students will watch the video, then tell what they noticed. They will then be asked to discuss what they wonder or are curious about. These questions will be recorded on a class chart or on the board. Students will then use mathematics to answer their own questions. Students will be given information to solve the problem based on need. When they realize they don't have the information they need, and ask for it, it will be given to them.

Task Description
The following 3-Act Task can be found at: http://www.estimation180.com/stackingcups.html

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

ACT 1:
Watch the video, (link: http://www.estimation180.com/stackingcups.html)
Ask students what questions they have about the video.

ACT 2:
What information would be useful to know here and how would you get it?/

Image 1: Dimensions of Styrofoam cup,
http://www.estimation180.com/uploads/1/3/8/8/13883834/stacking_cups_act_2a.jpg

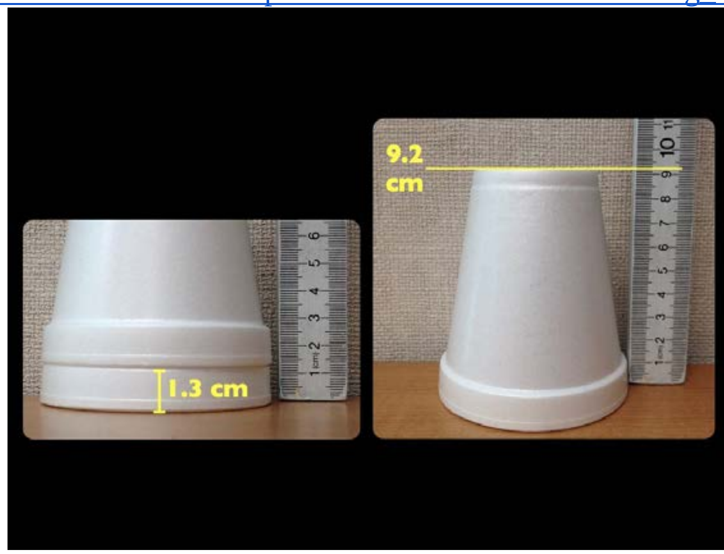
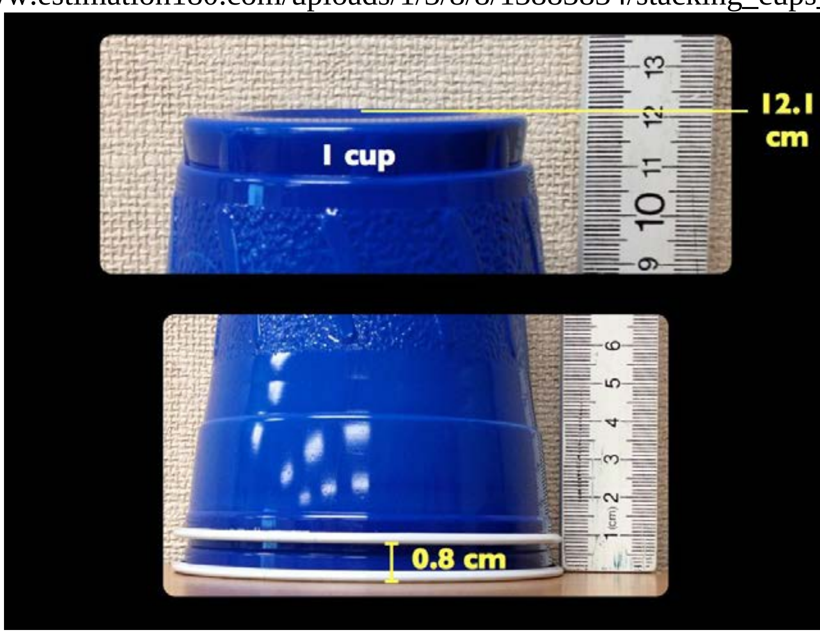


Image 2: Dimensions of plastic cups:
http://www.estimation180.com/uploads/1/3/8/8/13883834/stacking_cups_act_2b.jpg



With the information from Act 2, find out where the stacks of cups will tie.

- Students may work individually for 5-10 minutes to come up with a plan, test ideas, and problem-solve.
- After individual time, students may work in groups (3-4 students) to collaborate and solve this task together.

- ***Finished early?** Challenge students with a Sequel (*extension*).

ACT 3
Video: The answer, http://www.estimation180.com/stackingcups.html
If we weren't exactly right, what could account for the error?

ACT 4

Extension:

- **Create a Title:** How should we title this lesson so it captures the mathematics we used and where we used it?
- **The Sequel:** Invent 2 cups where their height will be equal at 100 cups.

Intervention:

- Provide students with cups to simulate the activity. Note: The measurements above may need to be altered due to manufacturer.
- Provide students with a table to complete.

Styrofoam Cups		*Solo Cups*	
# of Styrofoam Cups	**Height of Styrofoam Cups**	**# of Solo Cups**	**Height of Solo Cups**

Stacking Cups

Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate the result of the main question? Explain?		
Place an estimate that is too high and too low on the number line		
Low estimate	*Place an "x" where your estimate belongs*	High estimate

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:______________________________

Act 2 (con't)
Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Field Day

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

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MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
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7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

COMMON MISCONCEPTIONS:

- *Students may infer that only x and y can be used for variables. This may become problematic when students are given equations with different variables. To address this misconception, discuss with students which variables they would like to use within a certain context. Groups of students may have different ideas about which variables to use, but the idea that any variable will work, as long as it is defined in the beginning, is the big idea here.*
- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

ESSENTIAL QUESTIONS:

- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?

MATERIALS:

- Colored pencils
- Straightedge
- Graphing calculator *(optional)*
- Graph paper http://incompetech.com/graphpaper
- Copies of task for students

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use a system of linear functions to solve a problem situation. This task could be modified to reflect other sports, teams, and players that are favorites of your students.

DIFFERENTIATION:

Extension:

- How many feet would your opponent need as a head start in order for the two of you to tie in the race? Show your work and explain your thinking as you did in the original problem.

Intervention/Scaffolding:

- Prompt struggling students with guiding questions. Group students in a way that promotes cooperative learning.

Field Day

You have been selected by the members of your eighth-grade team (Team A) to participate in a one-mile race on Field Day. Another student from Team B will race against you. You know from some trials in PE that you run at an average rate of 12 feet per second. Since the student from Team B runs 10 feet per second, you have been asked to let him have a 1000-ft. head start. If both of you maintain the estimated rates (12 feet per second and 10 feet per second), would you be able to beat your opponent? Use more than one method to justify your conclusion.

Solution

The total distance run in x seconds by the student from Team B will be 1000 + 10x. The total distance run in x seconds by the student from Team A will be 12x. The time required for the distances to be the same is found by solving 1000 + 10x = 12x. This yields 2x = 1000, or x = 500 seconds. However, after 500 seconds, both students would have run 6000 feet. Since this is greater than one mile (5280 feet), the race would already have ended. Therefore, it is not possible for the student from Team A to win under these circumstances.

Using graphing calculator technology, enter $y_1 = 1000 + 10x$ and $y_2 = 12x$.
Students will need to realize that the y value tells the distance from the start line and that the distance of the runner from Team B from the starting line is always greater than the distance of the runner from Team A from the starting line until the time is 500 seconds. At that point the distances are the same. (Note that although an appropriate window for this problem situation might be $x_{min} = 0$, $x_{max} = 500$, $y_{min} = 0$, and $y_{max} = 6000$, the graphs are better compared with settings such as $x_{min} = 400$, $x_{max} = 550$, $y_{min} = 5000$, and $y_{max} = 6200$. In tracing along either of the lines, the student can determine that the y value exceeds 5280 feet before the point of intersection of the two lines. This means that the runner from Team A cannot pass the runner from Team B in less than a mile.)

A sample of Georgia eighth-grade student work is shown on the next page.

field day

answer - no. you could not beat your opponent.

Team a

$12x = y$

x	y
12(400)	4800
12(410)	4920
12(420)	6040
12(430)	5160

Team B

$10x + 1000 = y$

x	y
10(400)+1000	5000
10(410)+1000	5100
10(420)+1000	5200
10(430)+1000	5300

at 430 seconds, runner B has already gone over a mile, and runner a has not even reached a mile yet.

one mile = 5260 ft.

Team a-

12 represents how many feet the person can run in one second.

x represents how many seconds they have been running

y represents how far the individual has ran in x seconds

Team B-

10 represents how many feet the person can run in one second.

x represents how many seconds the runner has been running.

1000 represents how many feet ahead runner B was at the beginning.

y represents how far the individual has ran in x seconds.

Field Day

You have been selected by the members of your eighth-grade team (Team A) to participate in a one-mile race on Field Day. Another student from Team B will race against you. You are able to run 12 feet per second. Since the student from Team B runs 10 feet per second, you have been asked to let him have a 1000-ft. head start. If both of you maintain the estimated rates (12 feet per second and 10 feet per second), would you be able to beat your opponent? Use more than one method to justify your conclusion.

Free Throw Percentages

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

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MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHER NOTES:

This task has a nice sports connection to it. While reinforcing how averages are figured in basketball, the task requires students to dive deeper into the statistics of the sport. Students will write generalizations, creating algebraic expressions, review rational and irrational numbers, write and solve proportions. This task has a lot to offer students.

COMMON MISCONCEPTIONS:

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations. Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.*

ESSENTIAL QUESTIONS:

- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?

MATERIALS:

- Colored pencils
- Straightedge
- Graphing calculator *(optional)*
- Graph paper http://incompetech.com/graphpaper
- Copies of task for students

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use a system of linear functions to solve a problem situation. This task could be modified to reflect other sports, teams, and players that are favorites of your students.

DIFFERENTIATION:

Extension:

- Later in the season Wade is now hitting 84 percent of his free throws. If he has made 64 free throws, how many free throws has he attempted? Would it make sense for this answer to have a decimal place? Why or why not?

Intervention/Scaffolding:

- Prompt struggling students with guiding questions. Group students in a way that promotes cooperative learning. Note: Struggling students will need much guidance while doing this task. If you have a large number of struggling students in a class, you may want to do this task together as a class.

Free Throw Percentages

Imagine that you are sitting in front of a television watching the Miami Heat playing the Dallas Mavericks. Dwayne Wade drives the lane and is fouled. As he steps to the free throw line, the announcer states that "Wade is hitting 82 percent of his free throws this year." He misses the first shot, but makes the second. Later in the game, Dwayne Wade is fouled for the second time. As he approaches the free throw line, the announcer states that "Wade has made 78 percent of his free throws so far this year."

1. How are free throw percentages calculated?

Solution

Free throw percentages are calculated by dividing the total number of attempted shots into the number of successful shots.

2. What kind of numbers must we always be dealing with? Why?

Solution

We are dealing with positive rational numbers that are less than or equal to 1 because they cannot miss more shots than they attempt and if they were successful for every attempt the percentage would equal 1.

3. What algebraic relationships could you write to represent the two situations?

Solution

If x = the total number of attempted free throws for the first situation and y = the number of successful shots for the first situation, the first situation could be represented as either the proportion $\frac{y}{x}=\frac{82}{100}$***, or the equation*** $y = 0.82x$.

The second situation could be represented as either the proportion $\frac{y+1}{x+2}=\frac{78}{100}$***, or the equation*** $y = 0.78x + 0.56$.

Some students may want to let x – 2 = the first situation's attempted free throws with x = to the second situation's free throws instead. This would still be mathematically correct.

4. How many free throws has Dwayne Wade attempted so far this year?

Solution

This may be solved in several different ways. One possible method would be using substitution.

Since y = 0.82x and y = 0.78x + 0.56, we may substitute the first equation for y in the second equation. This will give 0.82x = 0.78x + 0.56. Subtracting 0.78x from both sides of the equation will yield 0.04x = 0.56. Dividing both sides of the equation by 0.04 will result in x = 14, which means that Dwayne Wade has attempted 14 free throws so far this year.

5. How many free throws has he made so far this year?

Solution

Once again, the students may work this in different ways. Most will substitute the value of x into one of the two equations such as, y = 0.82(14) or 11.48.
This result will most likely spark some good discussion about percentages in sports. To arrive at this answer must mean that the percentages were rounded to the nearest 0.01 and were not exact figures.

By substitution, students should see that $\frac{11}{14} \approx 0.7857142857$.

6. Are the answers you found in problems 4 and 5 unique? Why or why not?

Solution

Students should discover that using the given percentages, everyone found the same results because the only point that the two equations have in common is the point (14, 11.48).

7. How could you graphically defend your answer?

Solution

Students may either graph these equations on the same coordinate plane using paper and pencil or technology. It is suggested that they experience graphing by hand when they are first introduced to systems of equations when the solutions are nice and neat whole numbers that are easy to see on the graph and to check with the equations. As they become more familiar with how to determine the graphs, what the graphs mean, and when the numbers are not quite so pretty; it is appropriate to use available technology.

Free Throw Percentages

Imagine that you are sitting in front of a television watching the Miami Heat playing the Dallas Mavericks. Dwayne Wade drives the lane and is fouled. As he steps to the free throw line, the announcer states that "Wade is hitting 82 percent of his free throws this year." He misses the first shot, but makes the second. Later in the game, Dwayne Wade is fouled for the second time. As he approaches the free throw line, the announcer states that "Wade has made 78 percent of his free throws so far this year."

1. How are free throw percentages calculated?

2. What kind of numbers must we always be dealing with? Why?

3. What algebraic relationships could you write to represent the two situations?

4. How many free throws has Dwayne Wade attempted so far this year?

5. How many free throws has he made so far this year?

6. Are the answers you found in problems 4 and 5 unique? Why or why not?

7. How could you graphically defend your answer?

How Much Did They Cost?

[Back to Task Table]

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHER NOTES:

This task focuses on solving systems of equations algebraically, instead of graphically. This task is a nice task to allow students to decide the variables to use. Using ***t*** for tacos and ***b*** for burritos may "make sense," however, students may choose whatever variables make sense to them as long as they are defined.

COMMON MISCONCEPTIONS:

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

ESSENTIAL QUESTIONS:

- How do I decide which method would be easier to use to solve a particular system of equations?

MATERIALS:

- Copies of task for students

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use the elimination method to solve a system of linear equations.

DIFFERENTIATION:

Extension:

- Sales tax at Taco Town is 4%. What would be the new total costs for Mr. Nelson's two trips to Taco town? Including sales tax, how much does one taco cost now? Including sales tax, how much does one burrito cost now? Explain your reasoning. Challenge: Do you think you could modify your equations to include the sales tax?

Intervention/Scaffolding:

- Students requiring intervention/scaffolding should be given the opportunity to model the problem with manipulatives (possibly different colored tiles for burritos and tacos) and have a discussion about what would happen if the equations were multiplied by another number (scaled up). Why the solution would not change is a big idea in algebra and should be developed in students rather than told to them.

How Much Did They Cost?

Mr. Nelson went to Taco Town to get lunch for the eighth-grade teachers one day when they were having a grade-level meeting. He bought eight tacos and five burritos, and the total cost before tax was $13.27. On the day of the next grade-level meeting, he went back to Taco Town and got six tacos and seven burritos for a cost of $14.47 before tax. The teachers now want to pay Mr. Nelson, but Mr. Nelson doesn't remember how much one taco costs or how much one burrito costs.

Ms. Payne tells the teachers that there is enough information to figure out how much a taco costs and how much a burrito costs and that she will get one of her math students to do the calculation. She has chosen you to do the work. Find the cost of one taco and the cost of one burrito, showing your work in arriving at the answer. Provide a written explanation to present to the eighth-grade teachers so that they will understand how you solved the problem.

Solution

While mathematics teachers want to honor different solution paths, the intent is for the student to utilize the efficiency of solving a system of equations by elimination. Using substitution introduces fractions that make the calculations less straightforward than elimination. Likewise, to solve by graphing using a calculator requires putting the equations in slope-intercept form, resulting in fractions.

The two linear equations in the system are $8t + 5b = 13.27$ and $6t + 7b = 14.47$.

One of the first possible steps towards eliminating the t's could be to multiply the first equation by -3 and the second equation by 4. Thus, the original system has been replaced by the equivalent system:

$$-24t - 15b = -39.81$$
$$24t + 28b = 57.88$$

Adding the left and right sides of these equations yields $13b = 18.07$.
Dividing each side by 13 yields $b = \$1.39$.
Substituting this b value into the first equation produces $8t + 5(1.39) = 13.27$.
Multiplying gives $8t + 6.95 = 13.27$.
Subtracting 6.95 from each side results in $8t = 6.32$.
Dividing by 8 gives $t = .79$.

Thus the cost of each taco is $.79 cents and the cost of each burrito is $1.39.

Samples of Georgia eighth-grade student work are shown on the following pages.

Sample 1

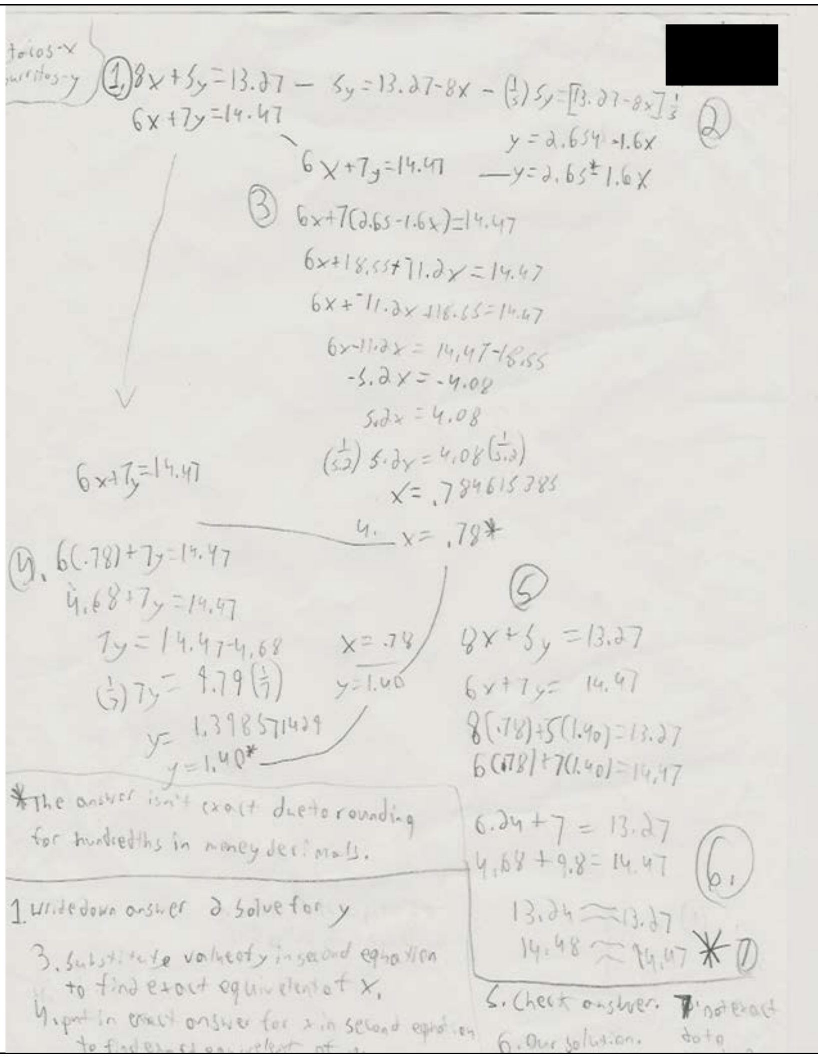
Teacher Commentary

The student correctly translates the problem situation into a system of equations. Choosing to solve the system by substitution, he clearly shows each step. Apparently he uses the calculator to do the arithmetic, and he recognizes that the answer is not exact because of rounding-off error. He shows checking the result in both equations.

The student should be encouraged to state that the variables represent the number of tacos and the number of burritos, rather than just putting x = tacos and y = burritos. In situations where there are two measurements associated with objects, it helps to clearly have in mind exactly what the variables represent. For example, in typical problems dealing with coins students must distinguish between the number of coins and the value of the coins.

The student's attempt at explaining the procedure to the eighth-grade teachers summarizes his steps. His first step of "Write down answer" is not clear. He needs to explain where the equations were obtained.

The student would benefit from solving the system by elimination and discovering the advantage of not introducing rounded off decimal forms for fractions. The student should also be encouraged to state the meaning of the solution to the system in terms of the original problem context.


[redacted]
8x + 5y = 13.27
6x + 7y = 14.47
y = 2.654 - 1.6x
6x+7(2.65-1.6x)=14.47
x = .78
y = 1.40
*The answer isn't exact due to rounding for hundredths in money decimals.
1. write down answer 2. solve for y
5. Check answer.
6. Our solution.

Sample 2

Teacher Commentary

The student efficiently solves the system correctly using elimination and makes a good attempt at explaining the process. The student could work towards more explicit information in the explanation. For example, the sentence "For that reason we put 8x + 5y = 13.27." does not fully explain how the equation was obtained. Also the student could have explained exactly what had to be done "…to change the equations so that one variable would cancel out."

x = tacos
y = burritos
8x+5y=13.27
6x+7y=14.47

24x + 15y = 39.81
-24x - 28y = -57.88
-13y = -18.07
y = 1.39

8x + 5(1.39) = 13.27
8x + 6.95 = 13.27 - 6.95
8x = 6.32
x = .79

burritos = 1.39
tacos = .79

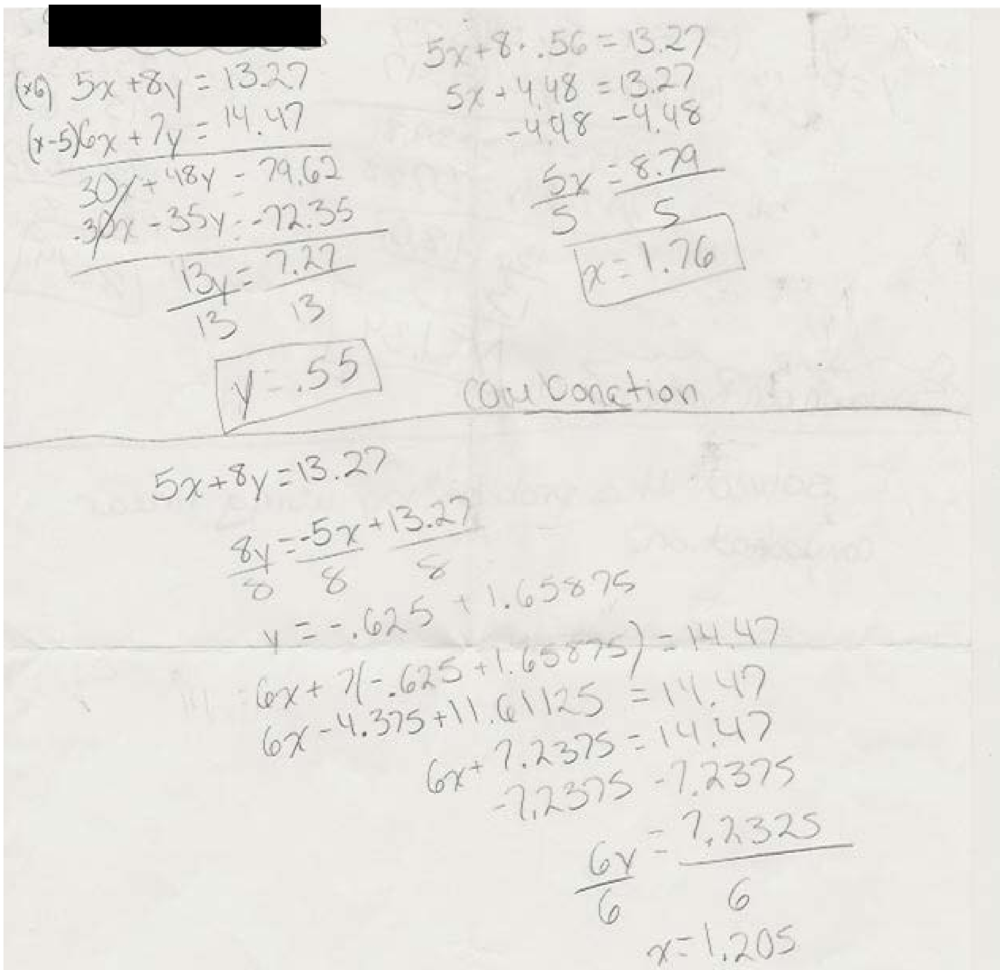
e chose to use linear combinations. First we gave the tacos
d burrito a variable. For the first equation we had to have 8 tacos
us 5 burritos to equal 13.27 we knew we had to figure out
w much each taco ~~and burrito~~ cost. For that reason we put
+5y=13.27. We also knew that he bought 6 tacos and 7 burritos
nother day and the total cost was 14.47. We had to change the
quations so that one variable would cancel out. Then we
added 15y and -28y and got -13y. Then we added the 2 totals
and got -18.07. We divided -13y and -18.07 by -13.
13y ended up to equal y and -18.07 ended up to be 1.39, which
was the cost for each burrito. Next we plugged in the cost
of y into the equation 8x + 5y = 13.27. Then we solved
for x which is .79 each taco.

Sample 3

Teacher Commentary

The student is to be commended for perseverance in problem-solving.

In her initial attempt, she does not identify what x and y represent. Consequently, it is not surprising that she has the coefficients on the variables reversed in one of the equations. This mistake led to wrong answers to the problem situation, even though the steps in solving the system she set up were correct.



[redacted]

(x6) $5x + 8y = 13.27$
(x-5) $6x + 7y = 14.47$
$30x + 48y = 79.62$
$-30x - 35y = -72.35$
$\frac{13y}{13} = \frac{7.27}{13}$
$y = .55$

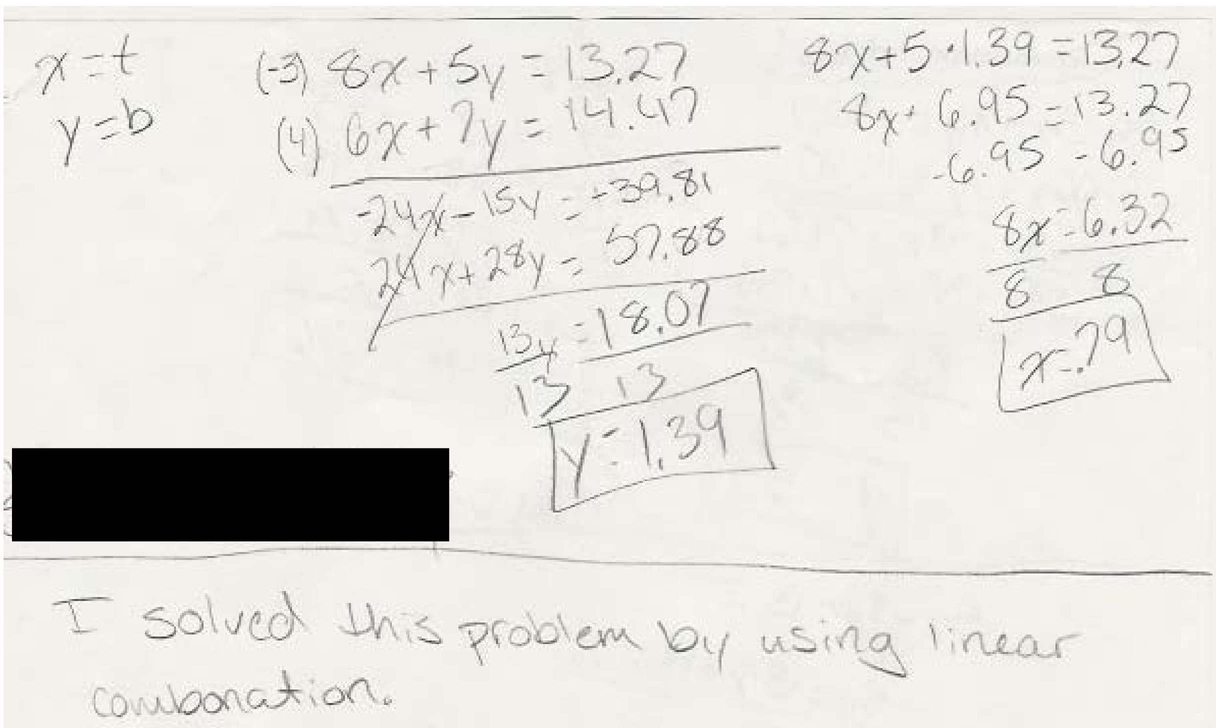
$5x + 8 \cdot .56 = 13.27$
$5x + 4.48 = 13.27$
$-4.48 \quad -4.48$
$\frac{5x}{5} = \frac{8.79}{5}$
$x = 1.76$

Correction

$5x + 8y = 13.27$
$\frac{8y}{8} = \frac{-5x}{8} + \frac{13.27}{8}$
$y = -.625 + 1.65875$
$6x + 7(-.625 + 1.65875) = 14.47$
$6x - 4.375 + 11.61125 = 14.47$
$6x + 7.2375 = 14.47$
$-7.2375 \quad -7.2375$
$\frac{6x}{6} = \frac{7.2325}{6}$
$x = 1.205$

Her next attempt uses substitution. Again the result is inaccurate, due to leaving off the variable x when she put the equation in y form.

In her final attempt, she shows in an abbreviated form (x = t and y = b) what she intends the variables to represent. She efficiently solves the system by elimination. She offers only a name for her solution path, but no explanation of how she set the system up. She needs examples of stating the results in terms of the original problem situation and explaining how to do a mathematical procedure to someone who does not understand.



Sample 4

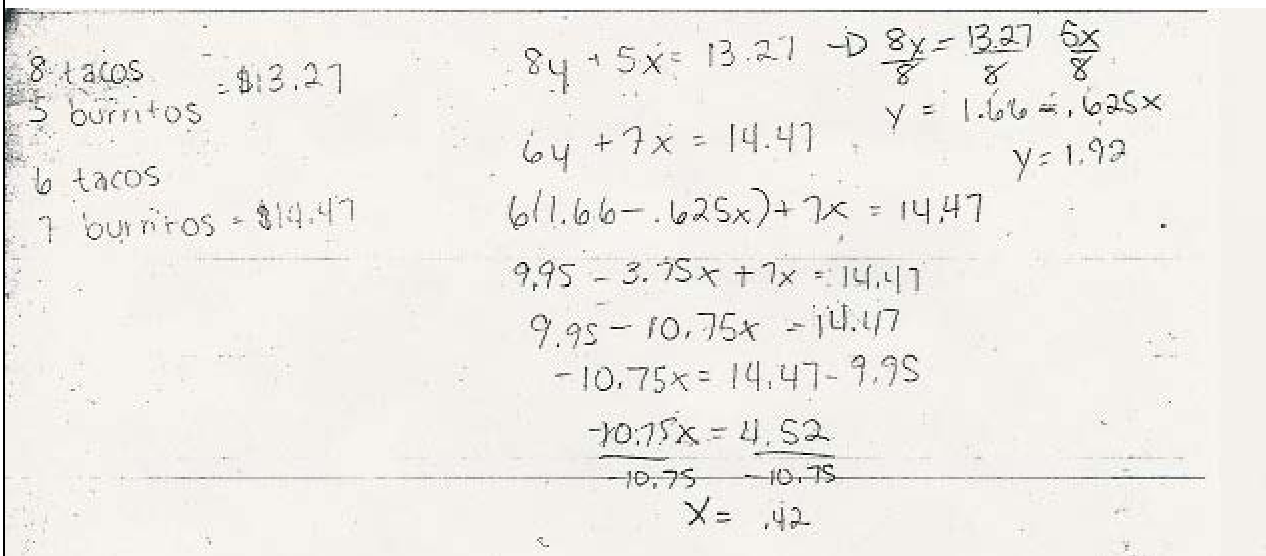
Teacher Commentary

The student shows organizing the given information on the left side and correctly translates this information into a system of equations. The student chooses to solve the system by elimination.

The rounding involved in this approach would only have resulted in a small error. However, the student's answer is not close to being accurate, because she combines -3.75x + 7x to get -10.75x rather than 3.27x.

The student should be encouraged to check the solution in both of the original equations. If she checks in the equation she used to calculate the y value, she will not catch her error. However, if she checks in the other equation, she should be able to see that a mistake has been made somewhere. There is no evidence that the student attempted to check her answer.

No interpretation of the results of the solution of the system is given. The student needs to state what the price of each taco is and what the price of each burrito is. She made no attempt to explain her process to the eighth-grade teachers, as was asked in the task.



8 tacos
5 burritos = $13.27

6 tacos
7 burritos = $14.47

$8y + 5x = 13.27 \rightarrow \frac{8y}{8} = \frac{13.27}{8} - \frac{5x}{8}$

$y = 1.66 - .625x$

$6y + 7x = 14.47$

$y = 1.92$

$6(1.66 - .625x) + 7x = 14.47$

$9.95 - 3.75x + 7x = 14.47$

$9.95 - 10.75x = 14.47$

$-10.75x = 14.47 - 9.95$

$\frac{-10.75x}{-10.75} = \frac{4.52}{-10.75}$

$x = .42$

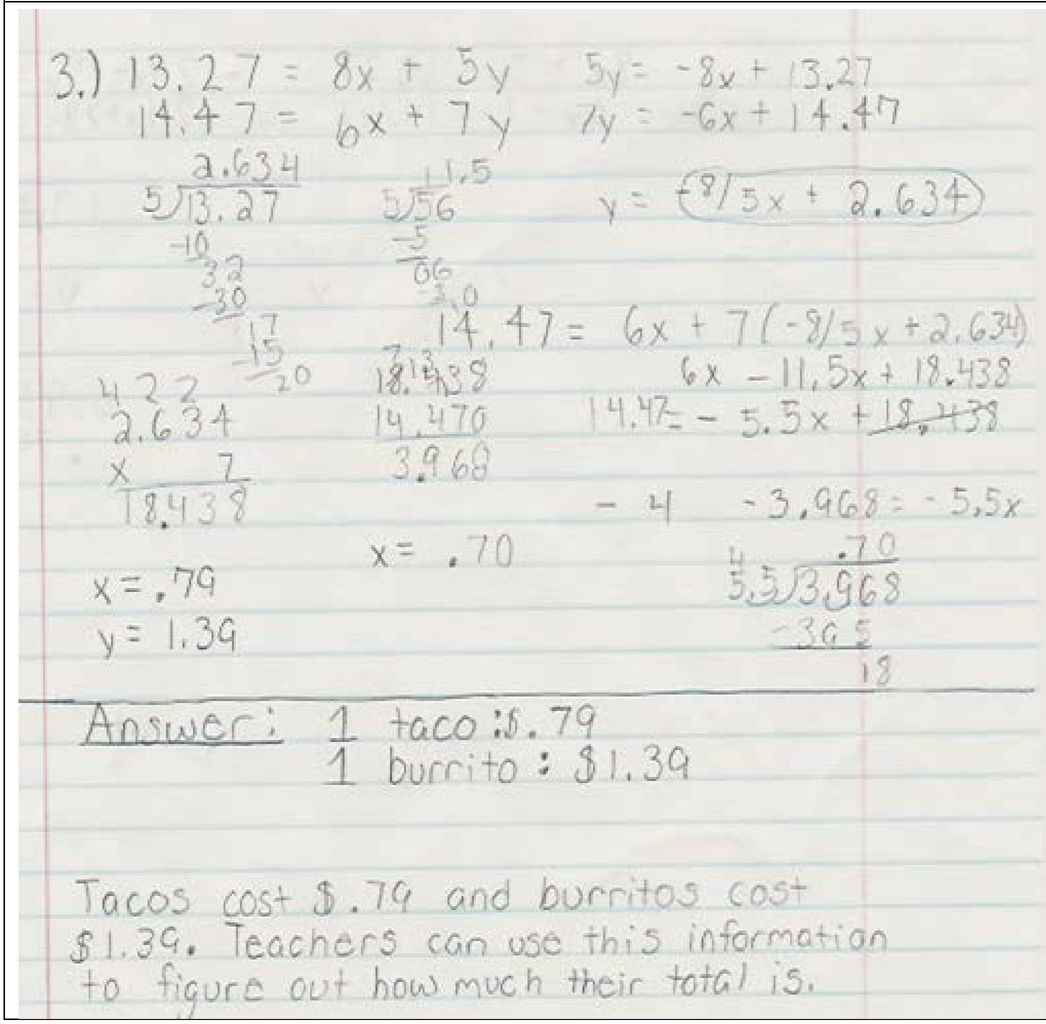
Sample 5

Teacher Commentary

The student initially chose to use substitution to solve the system. Due to arithmetic mistakes the answer obtained for the price of a taco was incorrect.

One mistake appears in the division of 13.27 by 5, where the student shows one instead of two when subtracting 30 from 32. Thus, the 2.634 quotient should have been 2.654. In dividing 3.968 by 5.5, the student misses the basic multiplication fact 7 x 5, indicating it is 45 instead of 35. If the student's arithmetic had been correct, then the result would have been that the price of a taco would have been 70 cents. However, the student shows that the price is 79 cents.

This conclusion appears to be the result of further work on her next page.


3.) 13.27 = 8x + 5y   5y = −8x + 13.27
14.47 = 6x + 7y   7y = −6x + 14.47

y = −8/5x + 2.634

14.47 = 6x + 7(−8/5x + 2.634)
6x − 11.5x + 18.438
14.47 = − 5.5x + 18.438
− 4   −3.968 = −5.5x

x = .70

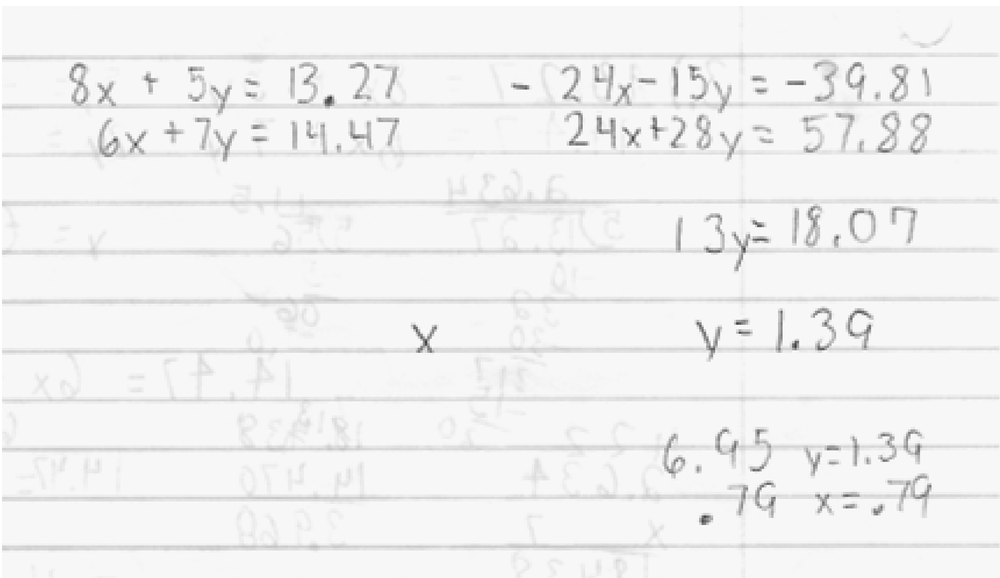
x = .79
y = 1.39

Answer: 1 taco : $.79
1 burrito : $1.39

Tacos cost $.79 and burritos cost $1.39. Teachers can use this information to figure out how much their total is.

While the student could have obtained accurate answers using the substitution method, the elimination method is less tedious for calculations. Interestingly, the student chose to use elimination method after having calculated the x value.

Since the student does not show dividing 18.07 by 13 to arrive at the y value, she may have used a calculator on this step. Apparently she also used the calculator to find the value of x after having gotten $1.39 for the price of each burrito.



Her work indicates that the total price of the burritos was $6.95, although she does not label this clearly. Evidently after finding the cost of each taco to be 79 cents using the calculator, she returned to the previous work and stated the answer without attempting to find the errors in arithmetic in her initial attempt.

The student is to be commended for stating her answer in a complete sentence. However, she does not explain to the eighth-grade teachers how she obtained her answer. She only states that the information can be used to figure out how much their total is. The explanation should include identifying what each variable stands for, what the equations in the system represent, and what steps were used in solving the system.

The student understands more than one method for solving systems and she can model situations with equations. She can state the result of the solution in the context of the original problem situation. She is developing a clearer sense of when it is preferable to use one solution method over another. Accurate results can be obtained with a calculator.

How Much Did They Cost?

Mr. Nelson went to Taco Town to get lunch for the eighth-grade teachers one day when they were having a grade-level meeting. He bought eight tacos and five burritos, and the total cost before tax was $13.27. On the day of the next grade-level meeting, he went back to Taco Town and got six tacos and seven burritos for a cost of $14.47 before tax. The teachers now want to pay Mr. Nelson, but Mr. Nelson doesn't remember how much one taco costs or how much one burrito costs.

Ms. Payne tells the teachers that there is enough information to figure out how much a taco costs and how much a burrito costs and that she will get one of her math students to do the calculation. She has chosen you to do the work. Find the cost of one taco and the cost of one burrito, showing your work in arriving at the answer. Provide a written explanation to present to the eighth-grade teachers so that they will understand how you solved the problem.

Fencing (Short Cycle Task)

Back to Task Table

Source: Balanced Assessment Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1086

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
6. Attend to precision.
7. Look for and make use of structure.

TEACHER NOTES:

This task offers students an opportunity to show their proficiency in solving a real world problem using their understandings of systems of equations. The task shows some student work, both scored and unscored, as well as a rubric for scoring, on the website below.

COMMON MISCONCEPTIONS:

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

ESSENTIAL QUESTIONS:

- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?

MATERIALS:

- See the task links

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Multiple Solutions*, is a Mathematics Assessment Project Assessment Task that can be found at the
website: http://map.mathshell.org/materials/tasks.php?taskid=369&subpage=apprentice

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1086

The scoring rubric can be found at the following link:
http://map.mathshell.org/materials/download.php?fileid=1087

DIFFERENTIATION:

Extension:

- Use the information you discovered in the Fencing Task regarding the price of a single fence post and a single fence panel to solve the following problem. The fence company will give you a discount for purchasing in bulk. They will charge $250 for 30 posts and $150 for 10 panels. If you need additional posts or panels you must purchase them individually. If you purchase 48 posts and 47 panels, using the bulk price for as much as you can, how much would it cost you?

Intervention/Scaffolding:

- Prompt struggling students with guiding questions.

Playing with Straws

[Back to Task Table]

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

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MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHER NOTES:

This task, with its puzzle-like feel, is engaging for students. The context, while a bit unbelievable, often will be soon "forgotten" as students begin to puzzle through the task. There is nice review in task as well – taking students back to angle relationships from Unit 1.

COMMON MISCONCEPTIONS:

- *Students may infer that only x and y can be used for variables. This may become problematic when students are given equations with different variables. To address this misconception, discuss with students which variables they would like to use within a certain context. Groups of students may have different ideas about which variables to use, but the idea that any variable will work, as long as it is defined in the beginning, is the big idea here.*

- *Students may infer that the variables should always be on the left side of the equations. This can be addressed early on (and is even being addressed in the elementary grades where this misconception begins to grow) by making sure students consistently see equations with variables on either side.*

- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations. Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.*

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

ESSENTIAL QUESTIONS:

How do I decide which method would be easier to use to solve a particular system of equations?

MATERIALS:

- Copies of task for students

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use a system of linear equations to solve a problem situation. Students will produce a written explanation of the method used to solve the problem.

DIFFERENTIATION:

Extension:

- Create your own straw design using at least three straws. Be sure you adhere to all angle properties. Use variables to label the angle measures similarly to the problem you just completed in the task, *Playing with Straws.* Give sufficient information to solve the problem using a system of equations.

Intervention/Scaffolding:

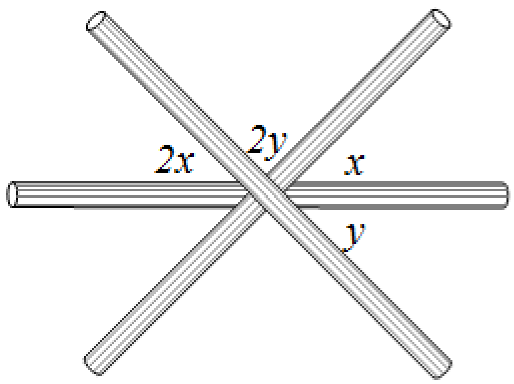
- Have students review angle relationships prior to beginning this task.

Playing with Straws

(Adapted from "Lunch Lines Continued" in the GPS Units)

Paul, Jane, Justin, Sarah, and Opal were finished with lunch and began playing with drink straws. Each one was making a line design using either 3 or 4 straws. Since they had just come from math class where they had been studying special angles, Paul pulled his pencil out of his bookbag and labeled some of the angles and lines. He then challenged himself and the others to find the values of x and y. He also double-dog dared them to find each of the angle measurements.

Explain to Jane, Justin, Sarah, and Opal how to find the values of x and y as well as how to determine each of the angle measurements for Paul's straw figure shown below.



Solution

There are three equations possible.

- ***Working clockwise from the top left, the first three angles add to*** $180°$***, therefore*** $3x+2y=180$.
- ***Likewise, the last three angles give*** $x+3y=180$.
- ***The final equation arises from vertical angles:*** $y=2x$.

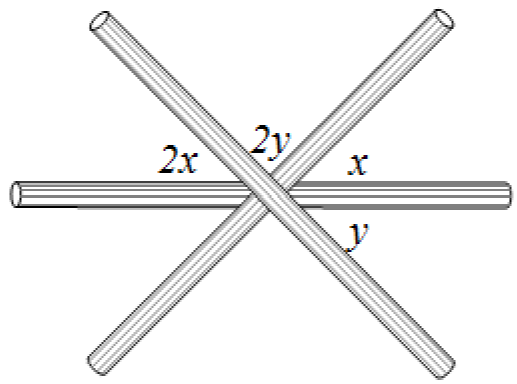
  Any two of these three are sufficient to solve the problem.
  Regardless of which combination of equations and whether they use substitution or elimination, students should end up at $x=\frac{180}{7}$ ***and*** $y=\frac{360}{7}$.

Playing with Straws

(Adapted from "Lunch Lines Continued" in the GPS Units)

Paul, Jane, Justin, Sarah, and Opal were finished with lunch and began playing with drink straws. Each one was making a line design using either 3 or 4 straws. Since they had just come from math class where they had been studying special angles, Paul pulled his pencil out of his book bag and labeled some of the angles and lines. He then challenged himself and the others to find the values of x and y. He also double-dog dared them to find each of the angle measurements.

Explain to Jane, Justin, Sarah, and Opal how to find the values of x and y as well as how to determine each of the angle measurements for Paul's straw figure shown below.



Planning a Party

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHER NOTES:

This task relates nicely to the Cara's Candles task and the DVDs task earlier in this unit. For those tasks as well as this, students could use substitution to solve the problem. This task also may be solved in other ways, such as numerically, with a table of values, making this task accessible to a wide range of students.

COMMON MISCONCEPTIONS:

- *Students may infer that only x and y can be used for variables. This may become problematic when students are given equations with different variables. To address this misconception, discuss with students which variables they would like to use within a certain context. Groups of students may have different ideas about which variables to use, but the idea that any variable will work, as long as it is defined in the beginning, is the big idea here.*

- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations. Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.*

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

- *Students may think that there is always one solution to systems of equations. This misconception should be addressed, as many of the others are, through contexts and contextual problems and sense-making. The contexts used in the problems we give students should engage students in reasoning and lead them to conclude when a system has no solutions, multiple solutions, or just one solution.*

ESSENTIAL QUESTIONS:

- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?

MATERIALS:

- Copies of task for students
- Calculators *(optional)*

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use the elimination method to solve a system of linear equations

DIFFERENTIATION:

Extension:

- Mr. Spain is planning to reward his students as well. He is purchasing pizzas which cost $8 each and drinks for $0.50 each. He needs 3 times as many drinks as pizzas. He only has $50 to spend. How many pizzas and drinks can he purchase? Would decimal answers make sense in this situation? Why or why not. Explain your answers.

Intervention/Scaffolding:

- Prompt struggling students with guiding questions.

Planning a Party

Ms. England is planning a party for students who have met their goal of reading seven books during the most recent nine weeks of school. Using a total of $112, she plans to get pizzas that cost $12 each and drinks that cost $0.50 each. If she purchases four times as many drinks as pizzas, how many of each should she buy? Show how you know.

Solution

Let x = the number of pizzas and y = the number of drinks. Then $12x + .5y = 112$ and $y = 4x$.

In the Cara's Candles task and the DVD Club task, the equations which modeled the problem situations were in the form $y = mx + b$ and could easily be solved by substitution by setting the two expressions that were equal to y equal to each other. In the Planning a Party task, one of the equations does not have the number one as the coefficient of y. Because one of the equations does have y isolated on one side of the equation, it is easy to use substitution. Thus, $12x + .5(4x) = 112$. Simplifying yields $12x + 2x = 112$, and $14x = 112$. Dividing by 14 results in $x = 8$. Since $y = 4x = 4(8) = 32$, the number of pizzas to buy is 8 and the number of drinks to buy is 32.

In translating the condition that there should be four times as many drinks as pizza, a very frequent reversal error occurs. To help students who translate $x = 4y$ to understand why this is incorrect, put in a particular value for the number of pizzas and find what the equation gives for the corresponding number of drinks.

A numerical solution to this problem would be to set up a table of values such as the one shown below:

Number of Pizzas	Number of Drinks Spent	Amount
1	4	$14
2	8	$28
3	12	$42
4	16	$56
5	20	$70
6	24	$84
7	28	$98
8	32	$112

Planning a Party

Ms. England is planning a party for students who have met their goal of reading seven books during the most recent nine weeks of school. Using a total of $112, she plans to get pizzas that cost $12 each and drinks that cost $0.50 each. If she purchases four times as many drinks as pizzas, how many of each should she buy? Show how you know.

Coefficients & Constant Ratios. . .They Mean Something?

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHER NOTES:

This task is the first in this unit to focus student attention explicitly on the coefficients of systems of equations and analyze them using the patterns they notice. Students will make sense of what the patterns are and what effect they have on the solutions (one solution, infinitely many solutions, or no solutions). Making sense of these ratios of coefficients and the patterns, then connecting them to the graphs will help students develop more efficient strategies for solving some systems of equations.

COMMON MISCONCEPTIONS:

- *Students may think that there is always one solution to systems of equations. This misconception should be addressed, as many of the others are, through contexts and contextual problems and sense-making. The contexts used in the problems we give* *students should engage students in reasoning and lead them to conclude when a system has no solutions, multiple solutions, or just one solution.*

- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations. Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.*

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

ESSENTIAL QUESTIONS:

- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?

MATERIALS:

- Copies of task for students
- Calculators *(optional)*

GROUPING:

- Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students are asked to identify patterns in the ratios of certain systems of equations and determine what connection these ratios have with the type of solutions the system has (infinitely many solutions, or no solutions).

Give each pair or small group of students a set of 10 cards. Students are to sort the cards based on the equations they see. Each card shows two equations and these two equations have a relationship. Students are to find the relationship and look for other cards with similar relationships. There are multiple pathways to determine the relationships. Students may decide to search for patterns on the cards and notice that the coefficients of one of the equations are multiples of the coefficients of the other equation, others may graph points on the cards using graph paper, a graphing calculator or **Desmos**. When students think they have matched cards with similar systems, they should write what the relationship for each set of cards on a post-it note. Once this is complete, students share their reasoning about the relationships and discuss the different solution methods graphed vs algebraic vs patterns. These can be very powerful discussions. Students may first share ideas about the cards with infinitely many solutions. This seems to be more accessible. For example, they may share that both equations are the same line since the equation for one can be transformed to the other equation by multiplying every term by the same number. Since they are the same equations, they must be the same lines, so the lines must be on top of one another.

As students determine what the graphs look like, teachers should guide the discussion to what those graphs mean in terms of solutions ***(lines that are collinear means infinitely many solutions; parallel lines mean no solutions)***.

Finally, to tie this all together, students should look at all of the methods and make an explicit statement concerning the coefficients that explains the difference between collinear (infinitely many solutions) and parallel lines (no solutions).

- ***If the ratio of the coefficients of the two equations is the same, but the ratio of the constant is different, the system has no solutions and the lines will be parallel.***
- ***If the ratio of the coefficients and the constants of the two equations are all the same ratio, the system has infinitely many solutions, and the lines are collinear.***

Teachers may wish to use the links to the following problems after completing this task. This will give students some additional practice before attempting the challenging performance task "What are the Coefficients" that follows.

http://www.openmiddle.com/system-of-equations-special-case-infinitely-many-solutions/

http://www.openmiddle.com/systems-of-equations-special-case-no-solution/

DIFFERENTIATION:

Extension:

- To make this more of a challenge, take out one of the cards from either solution type. Since this creates a situation where there are a different number of solutions   Create two of your own system of equations that have infinitely many solutions and/or no solutions. Write how you know these systems' solutions are what you say.

Intervention/Scaffolding:

- The number of cards may be limited to 6 (3 of each) in order to differentiate for your students. Students who struggle with the concept of no solution and infinitely many solutions will need much guidance while doing this task. If you have a large number of struggling students in a class, you may want to do this task together in small groups or as a whole class.

Coefficients & Constant Ratios. . .They Mean Something? Sorting Cards
***Solution**: Cut and shuffle cards prior to distributing to students.*

Infinitely Many Solutions	*No Solutions*
$2x - y = 4$ $6x - 3y = 12$	$2x - y = 4$ $6x - 3y = 3$
$15x + 25y = -20$ $-3x - 5y = 4$	$4x + y = 9$ $2x + y = -1$
$-x + 2y = 7$ $-2x + 4y = 14$	$3x + -y = 14$ $9x + -3y = 8$
$x - y = 3$ $-3x + 3y = -9$	$6x - 21y = -4$ $2x - 7y = 11$
$3x + 9y = 15$ $-x - 3y = -5$	$3x - 2y = 2$ $-12x + 8y = 3$

Coefficients & Constant Ratios. . .They Mean Something? Sorting Cards

$2x - y = 4$ $6x - 3y = 12$	$2x - y = 4$ $6x - 3y = 3$
$15x + 25y = -20$ $-3x - 5y = 4$	$4x + y = 9$ $2x + y = -1$
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$x - y = 3$ $-3x + 3y = -9$	$6x - 21y = -4$ $2x - 7y = 11$
$3x + 9y = 15$ $-x - 3y = -5$	$3x - 2y = 2$ $-12x + 8y = 3$

What Are the Coefficients?

Back to Task Table

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHER NOTES:

This assessment item may present quite a challenge for students. Prior to assigning this task teachers should develop examples with equations given in slope-intercept form and facilitate student understanding of systems of equations with no solutions, multiple solutions, and one solution. This can be done by giving students equations and having them convert them to standard form while looking for patterns. This task requires students to look for these patterns and use this knowledge to make decisions.

Students might enjoy doing this practice with applets provided on the TI-84. Press the "APPS" button and select "ALG1CH5." The screen will then show "Topics in Algebra l: 5. Linear Systems." Students can review using the tutorials under "Overview" and "Observations." The "Activities" section provides practice on determining whether systems are consistent, dependent, or inconsistent. They may choose to practice at the "Silver" or

the “Gold” level. Hints in the form of graphs of the given equations are available. Feedback is provided as to whether the responses are correct.
For extensions, similar questions to the one in “What Are the Coefficients?” can be asked which deal with the cases of infinitely many solutions or one solution. For example, for what value of c would the following system have infinitely many solutions:

$$4x - 5y = -7$$
$$-12x + 15y = c$$

Since the two equations would have to represent the same line in order to have infinitely many solutions, the equations must be equivalent. Thus, -7 must be multiplied by -3, resulting in c = 21

COMMON MISCONCEPTIONS:

- *Students may infer that only x and y can be used for variables. This may become problematic when students are given equations with different variables. To address this misconception, discuss with students which variables they would like to use within a certain context. Groups of students may have different ideas about which variables to use, but the idea that any variable will work, as long as it is defined in the beginning, is the big idea here.*

- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations. Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.*

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

- *Students may think that there is always one solution to systems of equations. This misconception should be addressed, as many of the others are, through contexts and contextual problems and sense-making. The contexts used in the problems we give students should engage students in reasoning and lead them to conclude when a system has no solutions, multiple solutions, or just one solution.*

ESSENTIAL QUESTIONS:

- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?

MATERIALS:

- Copies of task for students
- Calculators *(optional)*

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will use the elimination method to solve a system of linear equations

DIFFERENTIATION:

Extension:

- Create two of your own system of equations scenarios. Create one similar to Scenario A and one similar to Scenario B. You will solve your problems and then trade with a partner so that they may solve your problem.

Intervention/Scaffolding:

- Prompt struggling students with guiding questions. Group students in a way that promotes cooperative learning. Note: Students who struggle with the concept of no solution and infinitely many solutions will need much guidance while doing this task. If you have a large number of struggling students in a class, you may want to do this task together as a class.

What Are the Coefficients?

Scenario A:

Find *d* so that the following system of equations has no solution:

$$4x - 5y = -7$$
$$-12x + dy = 28$$

Explain the relationship of these 2 lines.

What would the graph look like?

Solution

Students should recognize that if the ratio of the x coefficients is the same as the ratio of the y coefficients, but the ratio of the constants is different, then the lines represented by the equations will be parallel. In this problem the y coefficient in the top equation should be multiplied by -3, resulting in a d value of 15 producing a system with no solutions.

Scenario B:

Find *c* and *d* so that the following system of equations has infinite solution:

$$-2x + 7y = -10$$
$$cx + dy = 20$$

What do you notice about the relationship of the coefficients respectfully?

What would the graph look like?

Solution

Students should recognize that if the ratio of the x coefficients is the same as the ratio of the y coefficients, but the ratio of the constants is the same, then the lines represented by the equations will be collinear. In this problem the x and y coefficients in the top equation should be multiplied by -2, resulting in a c value of 4 and a d value of -14 producing a system with no solutions.

<u>What Are the Coefficients</u>?

<u>Scenario A:</u>

Find *d* so that the following system of equations has no solution:

$$4x - 5y = -7$$
$$-12x + dy = 28$$

Explain the relationship of these 2 lines.

What would the graph look like?

<u>Scenario B:</u>

Find *c* and *d* so that the following system of equations has infinite solution:

$$-2x + 7y = -10$$
$$cx + dy = 20$$

What do you notice about the relationship of the coefficients respectfully?

What would the graph look like?

Cell Phone Plans

Back to Task Table

Which plan is best for you?

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHER NOTES:

In order for students to be successful, the following skills and concepts need to be maintained:

- MCC5.G.1, MCC5.G.2, MCC6.EE.5, MCC6.EE.6, MCC6.EE.7, MCC7.EE.4

COMMON MISCONCEPTIONS:

- *Students may infer that only x and y can be used for variables. This may become problematic when students are given equations with different variables. To address this misconception, discuss with students which variables they would like to use within a certain context. Groups of students may have different ideas about which variables to use, but the idea that any variable will work, as long as it is defined in the beginning, is the big idea here.*

- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations. Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.*

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*

- *Students may think that there is always one solution to systems of equations. This misconception should be addressed, as many of the others are, through contexts and contextual problems and sense-making. The contexts used in the problems we give students should engage students in reasoning and lead them to conclude when a system has no solutions, multiple solutions, or just one solution.*

- *Students may infer that the variables should always be on the left side of the equations. This can be addressed early on (and is even being addressed in the elementary grades where this misconception begins to grow) by making sure students consistently see equations with variables on either side.*

ESSENTIAL QUESTIONS:

- What does the solution to a system tell me about the answer to a problem situation?
- What does the geometrical solution of a system mean?
- How can I translate a problem situation into a system of equations?

MATERIALS:

- Copies of task for students
- Calculators *(optional)*

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will read and interpret the graph of a system of linear equations. Students will use their information to make real-world choices about cell phone plans.

DIFFERENTIATION:

Extension:

- Create two cell phone plans that would cost the same when 80 minutes were used. At least one plan must have a flat fee component per month. You may not charge more than $30 for the flat fee or more than $0.25 per minute. Write the equations for your plans and solve them showing your work.

Intervention/Scaffolding:

- Prompt struggling students with guiding questions.

Cell Phone Plans

Which plan is best for you?

The graph below represents the monthly rate of two cell-phone companies.

Which company offers the better plan and why? Defend your reasoning.

Answers will vary.

Comments:

Another option to using the graph provided would be for the students to actually research the rates of companies in your area. You might also have them research limited vs. unlimited data plans.

Guiding questions:

- At how many minutes, do these companies charge the same amount?

Solution

20 inutes

- During what interval does company T offer a better value?

Solution

Company T is a better value when the consumer uses less than 20 minutes. $0 < m < 20$

- When does company S offer a better value?

Solution

Company S is a better value when the consumer uses more than 20 minutes. $m > 20$

What does this graph which compares Company Z and Company K tell you?

Solution

Answers may vary. Examples: Company Z is always more expensive. The price plans never intersect.

Cell Phone Plans

Which plan is best for you?

The graph below represents the monthly rate of two cell-phone companies.

Which company offers the better plan and why? Defend your reasoning.

What does this graph, which compares Company Z and Company K, tell you?

Optimization Boomerang - FAL

Back to Task Table

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1241

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

TEACHER NOTES:

In order for students to be successful, the following skills and concepts need to be maintained:

- MCC5.G.1, MCC5.G.2, MCC6.EE.5, MCC6.EE.6, MCC6.EE.7, MCC7.EE.4

COMMON MISCONCEPTIONS:

- *Students may infer that only x and y can be used for variables. This may become problematic when students are given equations with different variables. To address this misconception, discuss with students which variables they would like to use within a certain context. Groups of students may have different ideas about which variables to* *use, but the idea that any variable will work, as long as it is defined in the beginning, is the big idea here.*

- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations. Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.*
- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*
- *Students may think that there is always one solution to systems of equations. This misconception should be addressed, as many of the others are, through contexts and contextual problems and sense-making. The contexts used in the problems we give students should engage students in reasoning and lead them to conclude when a system has no solutions, multiple solutions, or just one solution.*
- *Students may infer that the variables should always be on the left side of the equations. This can be addressed early on (and is even being addressed in the elementary grades where this misconception begins to grow) by making sure students consistently see equations with variables on either side.*
- *Students may interpret the constraints and variables incorrectly. This can be addressed by focusing students on each of the constraints and/or meanings of the variables individually. Student discussions of these constraints before diving into the task may also be helpful.*

ESSENTIAL QUESTIONS:

- How can I translate a problem situation into a system of equations?
- What does the solution to a system tell me about the answer to a problem situation?
- How can I interpret the meaning of a “system of equations” algebraically and geometrically?

MATERIALS:

- See the task links.

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the

lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Optimization Boomerang*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=207&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1241

Classifying Solutions to Systems of Equations - FAL

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Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1213

In this task, students will classify solutions to a pair of linear equations by considering their graphical representations.

STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This lesson uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

TEACHER NOTES:

In order for students to be successful, the following skills and concepts need to be maintained:

- MCC5.G.1, MCC5.G.2, MCC6.EE.5, MCC6.EE.6, MCC6.EE.7, MCC7.EE.4

COMMON MISCONCEPTIONS:

- *Students may infer that only x and y can be used for variables. This may become problematic when students are given equations with different variables. To address this misconception, discuss with students which variables they would like to use within a certain context. Groups of students may have different ideas about which variables to use, but the idea that any variable will work, as long as it is defined in the beginning, is the big idea here.*
- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations. Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.*
- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*
- *Students may think that there is always one solution to systems of equations. This misconception should be addressed, as many of the others are, through contexts and contextual problems and sense-making. The contexts used in the problems we give students should engage students in reasoning and lead them to conclude when a system has no solutions, multiple solutions, or just one solution.*
- *Students may infer that the variables should always be on the left side of the equations. This can be addressed early on (and is even being addressed in the elementary grades where this misconception begins to grow) by making sure students consistently see equations with variables on either side.*
- *Students may interpret the constraints and variables incorrectly. This can be addressed by focusing students on each of the constraints and/or meanings of the variables individually. Student discussions of these constraints before diving into the task may also be helpful.*
- *Students make an incorrect assumption about the multiplicative properties of zero. See **Fluency** above.*
- *Students may apply the rules for multiplying negative numbers incorrectly. This is most likely a result of students who have not made sense of multiplying integers. Rather, they have memorized rules that they do not understand and often will forget or mix up these memorized rules. See **Fluency** above.*

ESSENTIAL QUESTIONS:

- How do I graph a linear equation?
- What does the point of intersection mean?
- What is a system of equations?
- What does it mean to solve a system of linear equations?
- How do I decide which method would be easier to use to solve a particular system of equations?

TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Classifying Solutions to Systems of Equations,* is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=411&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1213

Culminating Task: Stained Glass Window

(Revised by Carrie Pierce, Lee County Schools)

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STANDARDS FOR MATHEMATICAL CONTENT:

Analyze and solve linear equations and pairs of simultaneous linear equations.

MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

MGSE8.EE.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

MGSE8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be 5 and 6.*

MGSE8.EE.8c Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

STANDARDS FOR MATHEMATICAL PRACTICE:

This task uses all of the practices with emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

TEACHER NOTES:

In order for students to be successful, the following skills and concepts need to be maintained:

- MCC5.G.1, MCC5.G.2, MCC6.EE.5, MCC6.EE.6, MCC6.EE.7, MCC7.EE.4

COMMON MISCONCEPTIONS:

- *Students may conclude that all equations should be in slope-intercept form. This misconception is often a result of students learning specific forms of equations separately with no connection to context. To address this misconception, begin by giving students a contextual situation and, through sense-making and reasoning, write the equations.*

Begin with the context, so the context determines the form to be used, rather than having students memorize the forms and try to place them within certain contexts.

- *Students may be confused about the difference between one-variable and two-variable equations and the differences in solving them – especially simultaneously. To address this misconception with students, have classroom discussions about the differences between one-variable and two-variable equations and how they are alike and different.*
- *Students may infer that the variables should always be on the left side of the equations. This can be addressed early on (and is even being addressed in the elementary grades where this misconception begins to grow) by making sure students consistently see equations with variables on either side.*

ESSENTIAL QUESTIONS:

- How do I graph a linear equation?
- What does the point of intersection mean?
- What is a system of equations?

MATERIALS:

- Copies of task and rubric for students
- Straightedge
- Graph paper http://incompetech.com/graphpaper
- Colored pencils

GROUPING:

- Individual/Partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION:

In this task, students will graph linear equations and explain the points of intersection. Students should be well-informed of the requirements and rubric that will be used for assessing their work.

DIFFERENTIATION:

Differentiation Ideas

- ***Choose the linear equations for students who may need assistance.***
- ***Have students graph using the slope-intercept form instead of graphing by a table of values.***
- ***Give students a completed stained glass graph and have them begin the activity from identifying the points of intersection.***

Stained Glass Window Rubric

Name________________

	8	**6**	**4**	**2**
Graphs of Lines	All lines are graphed correctly	At least 9 lines graphed correctly.	At least 7 lines graphed correctly.	Less than 7 lines graphed correctly.
#3: Proving the Solution	All 8 points are correct.	At least 6 points are correct.	At least 4 points correct.	Less than 4 points correct.
	4	**3**	**2**	**1**
#1 & #2: Points of Intersection & Equations	All 4 points match the equations	3 points match the equations	2 points match the equations	1 point matches the equation

TOTAL:

Stained Glass Window

What to do:

1. Circle 3 linear equations in each box below.
2. Write your equations over the T-charts and complete each T-chart with three ordered pairs. (You must choose a negative, zero, and a positive.) Remember, these are the solutions to your equation.
3. Graph these 12 equations on the coordinate plane on your graph paper.
4. Write each line's equation neatly on or beside its graph.
5. Use color pencils or crayons to color each section of the "window" that your graphs have created.

$x = -8$ $x = -5$ $x = -1$ $x = 2$ $x = 7$ $x = 9$	$y = -9$ $y = -5$ $y = -2$ $y = 1$ $y = 6$ $y = 8$	$y = x + 5$ $y = 2x - 7$ $y = 4x + 8$ $y = 2x + 18$ $y = 1/4\ x - 6$ $y = 1/2\ x - 3$	$y = -x - 9$ $y = -2x + 8$ $y = -1/3\ x - 3$ $y = -1/4\ x + 5$ $y = -2x$ $y = -x + 12$

Eq:____________ Eq:____________ Eq:____________ Eq:____________

x	y

x	y

x	y

x	y

Eq:____________ Eq:____________ Eq:____________ Eq:____________

x	y

x	y

x	y

x	y

Eq:____________ Eq:____________ Eq:____________ Eq:____________

x	y

x	y

x	y

x	y

Now that you have graphed all 12 of your lines and colored your stained glass window, we need to consider these as systems of equations. Follow the instructions below to investigate.

1. Choose 4 points of intersection on your window. List them below. We'll call them A, B, C, and D.

   A =____________, B =____________, C =____________, D=____________

2. Write the equations of the two lines that intersect at each point.

A	B	C	D

3. Prove that each point is a solution to both equations.

A	B	C	D
Equation 1	Equation 1	Equation 1	Equation 1

Equation 2	Equation 2	Equation 2	Equation 2

Stained Glass Window

What to do:

1. Circle 3 linear equations in each box below.
2. Write your equations over the T-charts and complete each T-chart with three ordered pairs. (You must choose a negative, zero, and a positive.) Remember, these are the solutions to your equation.
3. Graph these 12 equations on the coordinate plane on your graph paper.
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Eq:____________ Eq:____________ Eq:____________ Eq:____________

x	y

x	y

x	y

x	y

Eq:____________ Eq:____________ Eq:____________ Eq:____________

x	y

x	y

x	y

x	y

Eq:____________ Eq:____________ Eq:____________ Eq:____________

x	y

x	y

x	y

x	y

Now that you have graphed all 12 of your lines and colored your stained glass window, we need to consider these as systems of equations. Follow the instructions below to investigate.

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A	B	C	D

4. Prove that each point is a solution to both equations.

A	B	C	D
Equation 1	Equation 1	Equation 1	Equation 1

Equation 2	Equation 2	Equation 2	Equation 2

Technology Resources

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MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

https://www.desmos.com/calculator
https://www.illustrativemathematics.org/content-standards/8/EE/C/8

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

https://www.illustrativemathematics.org/content-standards/8/EE/C/8
http://nzmaths.co.nz/resource/weighing-time
http://nzmaths.co.nz/resource/pigs-goats-and-sheep

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

https://www.desmos.com/calculator
https://www.illustrativemathematics.org/content-standards/8/EE/C/8
http://nrich.maths.org/5438
http://nzmaths.co.nz/resource/pigs-goats-and-sheep