Lesson 1: Modeling Linear Relationships

Classwork

Example 1: Logging On

Lenore has just purchased a tablet computer, and she is considering purchasing an Internet access plan so that she can connect to the Internet wirelessly from virtually anywhere in the world. One company offers an Internet access plan so that when a person connects to the company's wireless network, the person is charged a fixed access fee for connecting *plus* an amount for the number of minutes connected based upon a constant usage rate in dollars per minute.

Lenore is considering this company's plan, but the company's advertisement does not state how much the fixed access fee for connecting is, nor does it state the usage rate. However, the company's website says that a 10-minute session costs $0.40, a 20-minute session costs $0.70, and a 30-minute session costs $1.00. Lenore decides to use these pieces of information to determine both the fixed access fee for connecting and the usage rate.

Exercises 1–6

1. Lenore makes a table of this information and a graph where number of minutes is represented by the horizontal axis and total session cost is represented by the vertical axis. Plot the three given points on the graph. These three points appear to lie on a line. What information about the access plan suggests that the correct model is indeed a linear relationship?

Number of Minutes	Total Session Cost (in dollars)
0	
10	0.40
20	0.70
30	1.00
40	
50	
60	
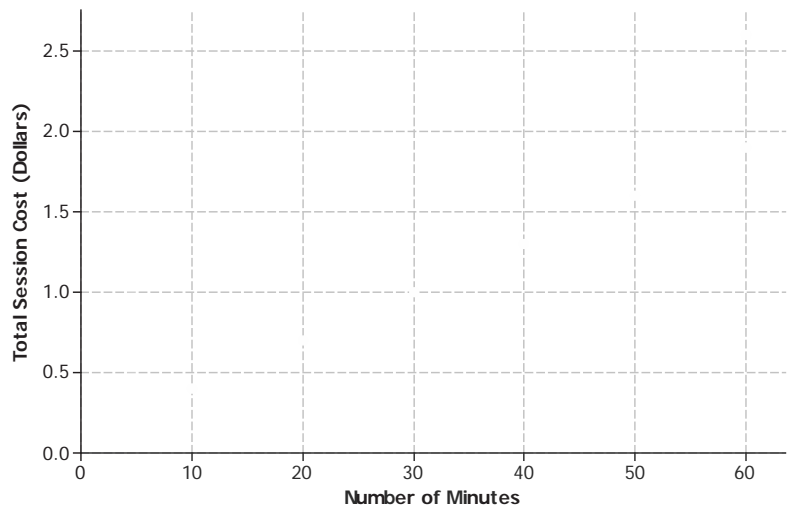
2. The rate of change describes how the total cost changes with respect to time.

    a. When the number of minutes increases by 10 (e.g., from 10 minutes to 20 minutes or from 20 minutes to 30 minutes), how much does the charge increase?

    b. Another way to say this would be the usage charge per 10 minutes of use. Use that information to determine the increase in cost based on only 1 minute of additional usage. In other words, find the usage charge per minute of use.

3. The company's pricing plan states that the usage rate is constant for any number of minutes connected to the Internet. In other words, the increase in cost for 10 more minutes of use (the value that you calculated in Exercise 2) is the same whether you increase from 20 to 30 minutes, 30 to 40 minutes, etc. Using this information, determine the total cost for 40 minutes, 50 minutes, and 60 minutes of use. Record those values in the table, and plot the corresponding points on the graph in Exercise 1.

4. Using the table and the graph in Exercise 1, compute the hypothetical cost for 0 minutes of use. What does that value represent in the context of the values that Lenore is trying to figure out?

5. On the graph in Exercise 1, draw a line through the points representing 0 to 60 minutes of use under this company's plan. The slope of this line is equal to the rate of change, which in this case is the usage rate.

6. Using x for the number of minutes and y for the total cost in dollars, write a function to model the linear relationship between minutes of use and total cost.

Example 2: Another Rate Plan

A second wireless access company has a similar method for computing its costs. Unlike the first company that Lenore was considering, this second company explicitly states its access fee is $0.15, and its usage rate is $0.04 per minute.

$$\text{Total Session Cost} = \$0.15 + \$0.04 \text{ (number of minutes)}$$

Exercises 7–16

7. Let x represent the number of minutes used and y represent the total session cost in dollars. Construct a linear function that models the total session cost based on the number of minutes used.

8. Using the linear function constructed in Exercise 7, determine the total session cost for sessions of 0, 10, 20, 30, 40, 50, and 60 minutes, and fill in these values in the table below.

Number of Minutes	Total Session Cost (in dollars)
0	
10	
20	
30	
40	
50	
60	

9. Plot these points on the original graph in Exercise 1, and draw a line through these points. In what ways does the line that represents this second company's access plan differ from the line that represents the first company's access plan?

MP3 download sites are a popular forum for selling music. Different sites offer pricing that depends on whether or not you want to purchase an entire album or individual songs à la carte. One site offers MP3 downloads of individual songs with the following price structure: a $3 fixed fee for a monthly subscription *plus* a charge of $0.25 per song.

10. Using x for the number of songs downloaded and y for the total monthly cost in dollars, construct a linear function to model the relationship between the number of songs downloaded and the total monthly cost.

11. Using the linear function you wrote in Exercise 10, construct a table to record the total monthly cost (in dollars) for MP3 downloads of 10 songs, 20 songs, and so on up to 100 songs.

12. Plot the 10 data points in the table on a coordinate plane. Let the x-axis represent the number of songs downloaded and the y-axis represent the total monthly cost (in dollars) for MP3 downloads.

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid $200. If 70 tickets are sold, the band will be paid $260.

13. Determine the rate of change.

14. Let x represent the number of tickets sold and y represent the amount the band will be paid in dollars. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

15. What flat fee will the band be paid for playing the concert regardless of the number of tickets sold?

16. How much will the band receive for each ticket sold?

Lesson Summary

A linear function can be used to model a linear relationship between two types of quantities. The graph of a linear function is a straight line.

A linear function can be constructed using a rate of change and an initial value. It can be interpreted as an equation of a line in which:

- The rate of change is the slope of the line and describes how one quantity changes with respect to another quantity.
- The initial value is the y-intercept.

Problem Set

1. Recall that Lenore was investigating two wireless access plans. Her friend in Europe says that he uses a plan in which he pays a monthly fee of 30 euro plus 0.02 euro per minute of use.

   a. Construct a table of values for his plan's monthly cost based on 100 minutes of use for the month, 200 minutes of use, and so on up to 1,000 minutes of use. (The charge of 0.02 euro per minute of use is equivalent to 2 euro per 100 minutes of use.)

   b. Plot these 10 points on a carefully labeled graph, and draw the line that contains these points.

   c. Let x represent minutes of use and y represent the total monthly cost in euro. Construct a linear function that determines monthly cost based on minutes of use.

   d. Use the function to calculate the cost under this plan for 750 minutes of use. If this point were added to the graph, would it be above the line, below the line, or on the line?

2. A shipping company charges a \$4.45 handling fee in addition to \$0.27 per pound to ship a package.

   a. Using x for the weight in pounds and y for the cost of shipping in dollars, write a linear function that determines the cost of shipping based on weight.

   b. Which line (solid, dotted, or dashed) on the following graph represents the shipping company's pricing method? Explain.

EUREKA MATH™
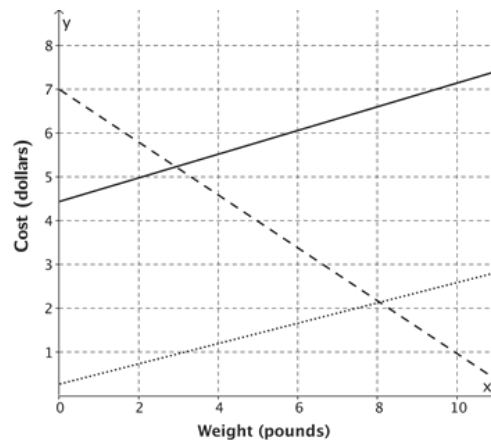
3. Kelly wants to add new music to her MP3 player. Another subscription site offers its downloading service using the following: Total Monthly Cost $= 5.25 + 0.30$ (number of songs).

   a. Write a sentence (all words, no math symbols) that the company could use on its website to explain how it determines the price for MP3 downloads for the month.

   b. Let x represent the number of songs downloaded and y represent the total monthly cost in dollars. Construct a function to model the relationship between the number of songs downloaded and the total monthly cost.

   c. Determine the cost of downloading 10 songs.

4. Li Na is saving money. Her parents gave her an amount to start, and since then she has been putting aside a fixed amount each week. After six weeks, Li Na has a total of $82 of her own savings in addition to the amount her parents gave her. Fourteen weeks from the start of the process, Li Na has $118.

   a. Using x for the number of weeks and y for the amount in savings (in dollars), construct a linear function that describes the relationship between the number of weeks and the amount in savings.

   b. How much did Li Na's parents give her to start?

   c. How much does Li Na set aside each week?

   d. Draw the graph of the linear function below (start by plotting the points for $x = 0$ and $x = 20$).