

Lesson 27: Nature of Solutions of a System of Linear Equations

Classwork

Exercises

Determine the nature of the solution to each system of linear equations.

1.
$$\begin{cases} 3x + 4y = 5 \\ y = -\frac{3}{4}x + 1 \end{cases}$$

2.
$$\begin{cases} 7x + 2y = -4 \\ x - y = 5 \end{cases}$$

3.
$$\begin{cases} 9x + 6y = 3 \\ 3x + 2y = 1 \end{cases}$$

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

4.
$$\begin{cases} 3x + 3y = -21 \\ x + y = -7 \end{cases}$$

5.
$$\begin{cases} y = \frac{3}{2}x - 1 \\ 3y = x + 2 \end{cases}$$

6.
$$\begin{cases} x = 12y - 4 \\ x = 9y + 7 \end{cases}$$

7. Write a system of equations with $(4, -5)$ as its solution.

Lesson Summary

A system of linear equations can have a unique solution, no solution, or infinitely many solutions.

Systems with a unique solution are comprised of two linear equations whose graphs have different slopes; that is, their graphs in a coordinate plane will be two distinct lines that intersect at only one point.

Systems with no solutions are comprised of two linear equations whose graphs have the same slope but different y -intercept points; that is, their graphs in a coordinate plane will be two parallel lines (with no intersection).

Systems with infinitely many solutions are comprised of two linear equations whose graphs have the same slope and the same y -intercept point; that is, their graphs in a coordinate plane are the same line (i.e., every solution to one equation will be a solution to the other equation).

A system of linear equations can be solved using a substitution method. That is, if two expressions are equal to the same value, then they can be written equal to one another.

Example:

$$\begin{cases} y = 5x - 8 \\ y = 6x + 3 \end{cases}$$

Since both equations in the system are equal to y , we can write the equation $5x - 8 = 6x + 3$ and use it to solve for x and then the system.

Example:

$$\begin{cases} 3x = 4y + 2 \\ x = y + 5 \end{cases}$$

Multiply each term of the equation $x = y + 5$ by 3 to produce the equivalent equation $3x = 3y + 15$. As in the previous example, since both equations equal $3x$, we can write $4y + 2 = 3y + 15$. This equation can be used to solve for y and then the system.

Problem Set

Determine the nature of the solution to each system of linear equations. If the system has a solution, find it algebraically, and then verify that your solution is correct by graphing.

1.
$$\begin{cases} y = \frac{3}{7}x - 8 \\ 3x - 7y = 1 \end{cases}$$

2.
$$\begin{cases} 2x - 5 = y \\ -3x - 1 = 2y \end{cases}$$

3.
$$\begin{cases} x = 6y + 7 \\ x = 10y + 2 \end{cases}$$

4.
$$\begin{cases} 5y = \frac{15}{4}x + 25 \\ y = \frac{3}{4}x + 5 \end{cases}$$

5.
$$\begin{cases} x + 9 = y \\ x = 4y - 6 \end{cases}$$

6.
$$\begin{cases} 3y = 5x - 15 \\ 3y = 13x - 2 \end{cases}$$

7.
$$\begin{cases} 6x - 7y = \frac{1}{2} \\ 12x - 14y = 1 \end{cases}$$

8.
$$\begin{cases} 5x - 2y = 6 \\ -10x + 4y = -14 \end{cases}$$

9.
$$\begin{cases} y = \frac{3}{2}x - 6 \\ 2y = 7 - 4x \end{cases}$$

10.
$$\begin{cases} 7x - 10 = y \\ y = 5x + 12 \end{cases}$$

11. Write a system of linear equations with $(-3, 9)$ as its solution.