Lesson 31: System of Equations Leading to Pythagorean Triples

Classwork

Exercises

1. Identify two Pythagorean triples using the known triple 3, 4, 5 (other than 6, 8, 10).

2. Identify two Pythagorean triples using the known triple 5, 12, 13.

3. Identify two triples using either 3, 4, 5 or 5, 12, 13.

Use the system $\begin{cases} x + y = \dfrac{t}{s} \\ x - y = \dfrac{s}{t} \end{cases}$ to find Pythagorean triples for the given values of s and t. Recall that the solution in the form of $\left(\dfrac{c}{b}, \dfrac{a}{b}\right)$ is the triple a, b, c.

4. $s = 4, t = 5$

5. $s = 7, t = 10$

6. $s = 1, t = 4$

7. Use a calculator to verify that you found a Pythagorean triple in each of the Exercises 4–6. Show your work below.

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Lesson Summary

A Pythagorean triple is a set of three positive integers that satisfies the equation $a^2 + b^2 = c^2$.

An infinite number of Pythagorean triples can be found by multiplying the numbers of a known triple by a whole number. For example, 3, 4, 5 is a Pythagorean triple. Multiply each number by 7, and then you have 21, 28, 35, which is also a Pythagorean triple.

The system of linear equations, $\begin{cases} x + y = \dfrac{t}{s} \\ x - y = \dfrac{s}{t} \end{cases}$, can be used to find Pythagorean triples, just like the Babylonians did 4,000 years ago.

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Problem Set

1. Explain in terms of similar triangles why it is that when you multiply the known Pythagorean triple 3, 4, 5 by 12, it generates a Pythagorean triple.

2. Identify three Pythagorean triples using the known triple 8, 15, 17.

3. Identify three triples (numbers that satisfy $a^2 + b^2 = c^2$, but a, b, c are not whole numbers) using the triple 8, 15, 17.

Use the system $\begin{cases} x + y = \dfrac{t}{s} \\ x - y = \dfrac{s}{t} \end{cases}$ to find Pythagorean triples for the given values of s and t. Recall that the solution, in the form of $\left(\dfrac{c}{b}, \dfrac{a}{b} \right)$, is the triple a, b, c.

4. $s = 2, t = 9$

5. $s = 6, t = 7$

6. $s = 3, t = 4$

7. Use a calculator to verify that you found a Pythagorean triple in each of the Problems 4–6. Show your work.