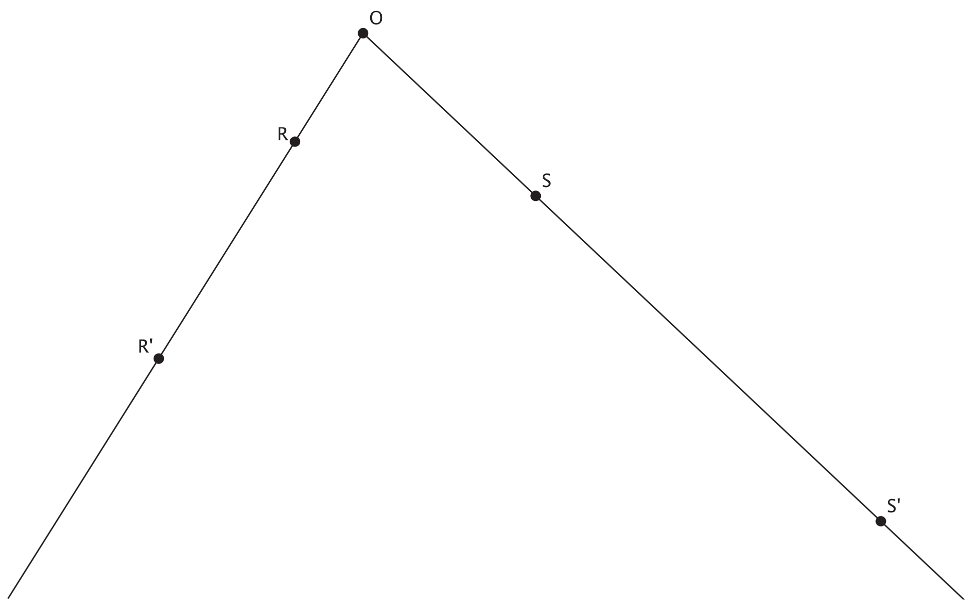
Lesson 4: Fundamental Theorem of Similarity (FTS)

Classwork

Exercise

In the diagram below, points R and S have been dilated from center O by a scale factor of $r = 3$.



a. If $|OR| = 2.3 \text{ cm}$, what is $|OR'|$?

b. If $|OS| = 3.5 \text{ cm}$, what is $|OS'|$?

c. Connect the point R to the point S and the point R' to the point S'. What do you know about the lines that contain segments RS and $R'S'$?

d. What is the relationship between the length of segment RS and the length of segment $R'S'$?

e. Identify pairs of angles that are equal in measure. How do you know they are equal?

Lesson Summary

THEOREM: Given a dilation with center O and scale factor r, then for any two points P and Q in the plane so that O, P, and Q are not collinear, the lines PQ and $P'Q'$ are parallel, where $P' = Dilation(P)$ and $Q' = Dilation(Q)$, and furthermore, $|P'Q'| = r|PQ|$.

Problem Set

1. Use a piece of notebook paper to verify the fundamental theorem of similarity for a scale factor r that is $0 < r < 1$.
   - ✓ Mark a point O on the first line of notebook paper.
   - ✓ Mark the point P on a line several lines down from the center O. Draw a ray, $\overrightarrow{OP}$. Mark the point P' on the ray and on a line of the notebook paper closer to O than you placed point P. This ensures that you have a scale factor that is $0 < r < 1$. Write your scale factor at the top of the notebook paper.
   - ✓ Draw another ray, $\overrightarrow{OQ}$, and mark the points Q and Q' according to your scale factor.
   - ✓ Connect points P and Q. Then, connect points P' and Q'.
   - ✓ Place a point, A, on the line containing segment PQ between points P and Q. Draw ray $\overrightarrow{OA}$. Mark point A' at the intersection of the line containing segment $P'Q'$ and ray $\overrightarrow{OA}$.

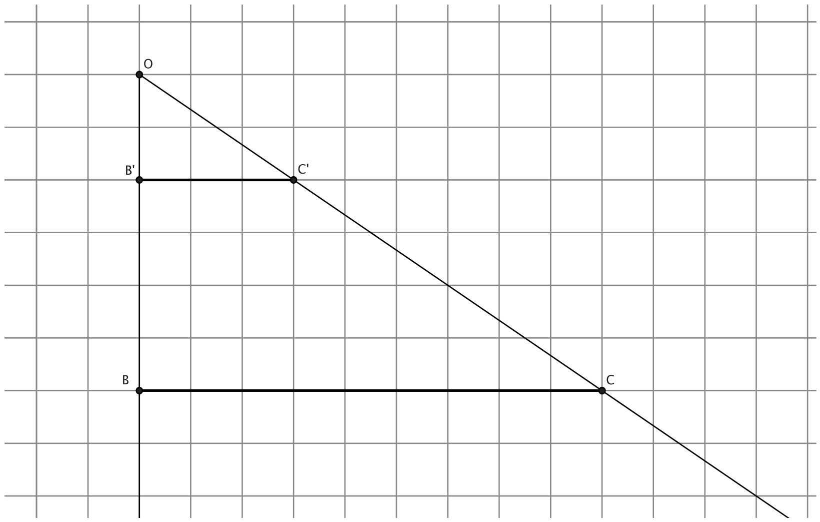
   a. Are the lines containing segments PQ and $P'Q'$ parallel lines? How do you know?

   b. Which, if any, of the following pairs of angles are equal in measure? Explain.
      i. $\angle OPQ$ and $\angle OP'Q'$
      ii. $\angle OAQ$ and $\angle OA'Q'$
      iii. $\angle OAP$ and $\angle OA'P'$
      iv. $\angle OQP$ and $\angle OQ'P'$

   c. Which, if any, of the following statements are true? Show your work to verify or dispute each statement.
      i. $|OP'| = r|OP|$
      ii. $|OQ'| = r|OQ|$
      iii. $|P'A'| = r|PA|$
      iv. $|A'Q'| = r|AQ|$

   d. Do you believe that the fundamental theorem of similarity (FTS) is true even when the scale factor is $0 < r < 1$? Explain.

2. Caleb sketched the following diagram on graph paper. He dilated points B and C from center O.



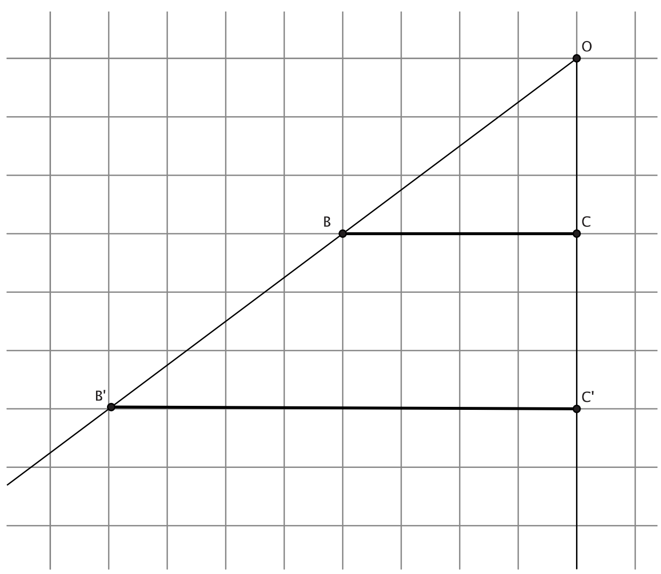
a. What is the scale factor r? Show your work.

b. Verify the scale factor with a different set of segments.

c. Which segments are parallel? How do you know?

d. Which angles are equal in measure? How do you know?

3. Points B and C were dilated from center O.



a. What is the scale factor r? Show your work.

b. If $|OB| = 5$, what is $|OB'|$?

c. How does the perimeter of triangle OBC compare to the perimeter of triangle $OB'C'$?

d. Did the perimeter of triangle $OB'C' = r \times$ (perimeter of triangle OBC)? Explain.