Lesson 5: Negative Exponents and the Laws of Exponents

Classwork

Definition: For any nonzero number x, and for any positive integer n, we define x^{-n} as $\frac{1}{x^n}$.

Note that this definition of negative exponents says x^{-1} is just the reciprocal, $\frac{1}{x}$, of x.

As a consequence of the definition, for a nonnegative x and all *integers* b, we get

$$x^{-b} = \frac{1}{x^b}$$

Exercise 1

Verify the general statement $x^{-b} = \frac{1}{x^b}$ for $x = 3$ and $b = -5$.

Exercise 2

What is the value of (3×10^{-2})?

Exercise 3

What is the value of (3×10^{-5})?

Exercise 4

Write the complete expanded form of the decimal 4.728 in exponential notation.

For Exercises 5–10, write an equivalent expression, in exponential notation, to the one given, and simplify as much as possible.

Exercise 5

$5^{-3} =$

Exercise 6

$\frac{1}{8^9} =$

Exercise 7

$3 \cdot 2^{-4} =$

Exercise 8

Let x be a nonzero number.

$x^{-3} =$

Exercise 9

Let x be a nonzero number.

$\frac{1}{x^9} =$

Exercise 10

Let x, y be two nonzero numbers.

$xy^{-4} =$

We accept that for nonzero numbers x and y and all integers a and b,

$$x^a \cdot x^b = x^{a+b}$$
$$(x^b)^a = x^{ab}$$
$$(xy)^a = x^a y^a.$$

We claim

$$\frac{x^a}{x^b} = x^{a-b} \qquad \text{for all integers } a, b.$$

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \qquad \text{for any integer } a.$$

Exercise 11

$$\frac{19^2}{19^5} =$$

Exercise 12

$$\frac{17^{16}}{17^{-3}} =$$

Exercise 13

If we let $b = -1$ in (11), a be any integer, and y be any nonzero number, what do we get?

Exercise 14

Show directly that $\left(\frac{7}{5}\right)^{-4} = \frac{7^{-4}}{5^{-4}}$.

Problem Set

1. Compute: $3^3 \times 3^2 \times 3^1 \times 3^0 \times 3^{-1} \times 3^{-2} =$
   Compute: $5^2 \times 5^{10} \times 5^8 \times 5^0 \times 5^{-10} \times 5^{-8} =$
   Compute for a nonzero number, a: $a^m \times a^n \times a^l \times a^{-n} \times a^{-m} \times a^{-l} \times a^0 =$

2. Without using (10), show directly that $(17.6^{-1})^8 = 17.6^{-8}$.

3. Without using (10), show (prove) that for any whole number n and any positive number y, $(y^{-1})^n = y^{-n}$.

4. Without using (13), show directly without using (13) that $\frac{2.8^{-5}}{2.8^7} = 2.8^{-12}$.

Equation Reference Sheet

For any numbers x, y [$x \neq 0$ in (4) and $y \neq 0$ in (5)] and any positive integers m, n, the following holds:	
$x^m \cdot x^n = x^{m+n}$	(1)
$(x^m)^n = x^{mn}$	(2)
$(xy)^n = x^n y^n$	(3)
$\frac{x^m}{x^n} = x^{m-n}$	(4)
$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	(5)
For any numbers x, y and for all whole numbers m, n, the following holds:	
$x^m \cdot x^n = x^{m+n}$	(6)
$(x^m)^n = x^{mn}$	(7)
$(xy)^n = x^n y^n$	(8)
For any nonzero number x and all integers b, the following holds:	
$x^{-b} = \frac{1}{x^b}$	(9)
For any numbers x, y and all integers a, b, the following holds:	
$x^a \cdot x^b = x^{a+b}$	(10)
$(x^b)^a = x^{ab}$	(11)
$(xy)^a = x^a y^a$	(12)
$\frac{x^a}{x^b} = x^{a-b} \qquad x \neq 0$	(13)
$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \qquad x, y \neq 0$	(14)